1. (30 p) The following matrix is defined in MATLAB:

\[
M = \begin{bmatrix}
6 & 9 & 12 & 15 & 18 & 21 \\
4 & 4 & 4 & 4 & 4 & 4 \\
2 & 1 & 0 & -1 & -2 & -3 \\
-6 & -4 & -2 & 0 & 2 & 4
\end{bmatrix}
\]

By hand (pencil and paper) write what will be displayed if the following commands are executed by MATLAB.

a) \( A = M(\{1, 3\}, \{2, 4\}) \)

b) \( B = M(:,\{1, 4:6\}) \)

c) \( C = M(\{2, 3\},:) \)

2. (35 p) The equations for the three loops in an electrical resistive network analysis problem can be written in a more general form as follows:

Flow around first loop: \( R_1(i_1 - i_2) + R_2(i_1 - i_3) = V_1 \)

Flow around second loop: \( R_3i_2 + R_4(i_2 - i_3) + R_1(i_2 - i_1) = V_2 \)

Flow around third loop: \( R_5i_3 + R_4(i_3 - i_2) + R_2(i_3 - i_1) = V_3 \)

Voltage sources and resistors are given as follows:

\( V_1 = 0 \text{ volt}, V_2 = 0 \text{ volt}, \text{ and } V_3 = 200 \text{ volt} \)

\( R_1 = 20 \Omega, R_2 = 10 \Omega, R_3 = 25 \Omega, R_4 = 10 \Omega, R_5 = 30 \Omega \)

Write a script M-file to find the currents \( i_1, i_2, i_3 \).

3. (35 p) A model for exponential growth of a quantity is given by:

\[
A(t) = A_0e^{kt}
\]

where \( A(t) \) and \( A_0 \) are the quantity at time \( t \) and time 0, respectively, and \( k \) is a constant unique to the specific application. Write a MATLAB user-defined function that uses this model to predict the quantity \( A(t) \) at time \( t \) from knowing \( A_0 \) and \( A(t_1) \) at some other time \( t_1 \). For function name and arguments use \( At = \text{expG}(A0, At1, t1, t) \), where the output argument \( At \) corresponds to \( A(t) \), and the input arguments \( A0, At1, t1, t \) correspond to \( A_0, A(t_1), t_1, \text{ and } t \), respectively. Use the function file in the Command Window for the following case:

The number of units produced was 67 million in the year 1980 and 79 million in 1986. Estimate the number of units produced in 2000.
1. (30 p)

```
>> M=[6:3:21;4,4,4,4,4,4;2:x1:x3;x6:2:4]
M =
6   9  12  15  18  21
4   4   4   4   4   4
2 1 0 -1 -2 -3
-6 -4 -2 0  2  4
>> A=M([1,3],[2,4])
A =
9  15
1 -1
>> B=M(:,[1,4:6])
B =
6  15  18  21
4   4   4   4
2 -1 -2 -3
-6  0  2   4
>> C=M([2,3],:)
C =
4  4  4  4  4  4
2  1  0 -1 -2 -3
```

2. (35 p) The three equations can be rewritten in matrix form \( [A][x] = [B] \)

\[
\begin{bmatrix}
(R_1 + R_2) & -R_1 & -R_2 \\
-R_1 & (R_1 + R_4 + R_3) & -R_4 \\
-R_2 & -R_4 & (R_2 + R_4 + R_5)
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
i_3
\end{bmatrix}
= 
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix}
\]

% M-file to solve four linear equations (array division)
% Define variables with the values of the V’s and R’s
V1=0; V2=0; V3=200;
R1=20; R2=10; R3=25; R4=10; R5=30;
A=[(R1+R2) -R1 -R2; -R1 (R1+R4+R3) -R4; -R2 -R4 (R2+R4+R5)];
% Right-hand side vector of B
B=[V1; V2; V3];
% Solving by using left division \( X = A \backslash B \)
I=A\B

3. (35 p)

To use the exponential growth model, the value of the constant \( k \) has to be determined first by solving for \( k \) in terms of \( A_0 \), \( A(t_1) \), and \( t_1 \):

\[
k = \frac{1}{t_1} \ln \frac{A(t_1)}{A_0}
\]

Once \( k \) is known, the model can be used to estimate the number of units produced at any time.
The user-defined function that solves the problem is:

```matlab
% Function definition line
function At = expG (A0, At1, t1, t)
% expG calculates exponential growth
% Input arguments are:
% A0: Quantity at time zero.
% At1: Quantity at time t1.
% t1: The time t1.
% t: time t.
% Output argument is:
% At: Quantity at time t.
k = log(At1/A0)/t1; % Determination of k
At=A0*exp(k*t); % Determination of A(t) (Assignment of value to output variable).
```

Once the function is saved, it will be used in the Command Window to solve the case. For the case given in the problem \(A_0 = 67\), \(A(t_1) = 79\), \(t_1 = 6\), and \(t = 20\):

**Command Window:**

```matlab
>> expG (67, 79, 6, 20)
ans =
   116.03
```

(Estimation of the number of units produced in the year 2000 based on the model).