1. (35 p) The amount $M$ of material present in the production tank depends on the rate that the material is consumed by the production process and on the rate that the material enters the production tank, where the rate at which the material is consumed is proportional to the amount present in the production tank. A differential equation for $M$ is:

$$\frac{dM}{dt} = -kM + p$$

where $k$ is the proportionality constant and $p$ is the rate that the material is supplied into the production tank. Solve this differential equation with the initial condition $M=M_0$ at $t=0$ using MATLAB’s symbolic capability.

2. (35 p) A company must purchase five kinds of material. The following table gives the price the company pays per ton for each material, along with the number of tons purchased in the months of May, June, and July:

<table>
<thead>
<tr>
<th>Material</th>
<th>Price ($/ton)</th>
<th>Quantity purchased (tons)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>May</td>
<td>June</td>
</tr>
<tr>
<td>1</td>
<td>300</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>550</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>250</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>500</td>
<td>4</td>
</tr>
</tbody>
</table>

Use MATLAB to answer the following questions:

a) What is the total spent in May, June and July?

b) What is the total spent on each material in the three-month period?

c) What is the total spent on all materials in the three-month period?

3. (35 p) The interest rate-Net Present Worth ($NPW$) data for a typical nonsimple investment are given for a graphical solution to the rate-of-return (ROR) problem as follows:

<table>
<thead>
<tr>
<th>Interest rate, $i$ (%)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>$NPW$ ($)</td>
<td>5928</td>
<td>6088</td>
<td>5840</td>
<td>5379</td>
<td>4815</td>
<td>4209</td>
<td>3597</td>
<td>2997</td>
<td>2422</td>
<td>1875</td>
<td>875</td>
<td>-3</td>
<td>-772</td>
</tr>
</tbody>
</table>

Use MATLAB to fit linear, quadratic, and cubic equation for the data, and plot them on the same graph.
1. First develop a hand solution to find $M$:

$$\frac{dM}{(-kM + p)} = dt$$

$$\int_0^M \frac{dM}{(-kM + p)} = \int_0^t dt$$

$$\frac{1}{-k} \left[ ln(-kM + p) \right]_0^M = \frac{1}{-k} \left[ ln(-kM + p) - ln(p) \right] = t$$

$$\left[ ln\left(\frac{-kM + p}{p}\right) \right] = -kt$$

$$\left(\frac{-kM + p}{p}\right) = e^{-kt}$$

$$M = \frac{p}{k} - \frac{p}{k} e^{-kt}$$

Secondly develop a MATLAB solution by using symbolic mathematics as follows:

```matlab
% Define symbolic variables
>> syms M k p t
% Use the dsolve command to solve the differential equation with the initial condition M=0 at t=0
>> Mt=dsolve('DM=-k*M+p','M(0)=0')
Mt=
p/k-p/k*exp(-k*t)
```

2. MATLAB session to answer the questions is:

```matlab
>> price=[300,550,400,250,500];
% Create 5x3 matrix (monthly_expenses) containing the amounts spent on each item for each month
>> quantity=[5,4,6;3,2,4;6,5,3;5,4;2,4,3];
>> monthly_expenses=[price'.*quantity(:,1),price'.*quantity(:,2),price'.*quantity(:,3)]
monthly_expenses=
    1500 1200 1800
    1650 1100 2200
    2400 2000 1200
    750 1250 1000
    1000 2000 1500
% Solution of Part (a)
>> May_expenses=sum(monthly_expenses(:,1))
    May_expenses =
    7300
>> June_expenses=sum(monthly_expenses(:,2))
    June_expenses =
    7550
>> July_expenses=sum(monthly_expenses(:,3))
    7700
% Solution of Part (b)
>> three_month_total=sum(monthly_expenses')
```
three_month_total =
4500 4950 5600 3000 4500
% Solution of Part (c)
>> total=sum(three_month_total)
total =
22550

3.
Create the MATLAB solution in an M-file (script file), then run it in the command environment:

% Fit linear, quadratic and cubic equation for the i-NPW data
interest_rate=[10 20 30 40 50 60 70 80 90 100 120 140 160];
net_present_worth=[5928 6088 5840 5379 4815 4209 3597 2997 2422 1875 875 -3 -772];
new_interest_rate=0:200;
% Fit linear equation for the data
newf1=polyval(polyfit(interest_rate,net_present_worth,1),new_interest_rate);
% Fit quadratic equation for the data
newf2=polyval(polyfit(interest_rate,net_present_worth,2),new_interest_rate);
% Fit cubic equation for the data
newf3=polyval(polyfit(interest_rate,net_present_worth,3),new_interest_rate);
% Plot fitting polynomials and the data on the same graph
plot(interest_rate,net_present_worth,'o',new_interest_rate,newf1, new_interest_rate,newf2,....
new_interest_rate,newf3
title('Fit of i-NPW data')
xlabel('Interest Rate, %')
ylabel('Net Present Worth(NPW), $')
legend('Data', 'Linear Fit', 'Quadratic Fit', 'Cubic Fit')