1. **(30 p)** Given a future cash flow \( F \) (future value) to be received in \( N \) years at an annual interest rate \( i \), will be discounted to the present time to find the present worth \( P \) (present value) of this future amount according to the equation given below:

\[
P = F (1 + i)^{-N}
\]

Solve this equation for \( i \) to find annual interest rate giving the present worth of future cash flow to be received after \( N \) years using MATLAB’s symbolic capability.

2. **(35 p)** The following table shows the hourly wages, hours worked, and output (number of widgets produced) in one week for five widget makers:

<table>
<thead>
<tr>
<th></th>
<th>Worker 1</th>
<th>Worker 2</th>
<th>Worker 3</th>
<th>Worker 4</th>
<th>Worker 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours worked</td>
<td>40</td>
<td>43</td>
<td>37</td>
<td>50</td>
<td>45</td>
</tr>
<tr>
<td>Output (widgets)</td>
<td>1000</td>
<td>1100</td>
<td>1000</td>
<td>1200</td>
<td>1100</td>
</tr>
<tr>
<td>Hourly wage ($)</td>
<td>5</td>
<td>5.50</td>
<td>6.50</td>
<td>6</td>
<td>6.25</td>
</tr>
</tbody>
</table>

Use MATLAB to answer the following questions:

a) How much did each worker earn in the week?
b) What is the total salary amount paid out?
c) How many widgets were made?
d) What is the average cost to produce one widget?
e) How many hours does it take to produce one widget on average?
f) Assuming that the output of each worker has the same quality, which worker is the most efficient? Which is the least efficient?

3. **(35 p)** An electronic company produces transistors, resistors, and computer chips. Each transistor requires four units of copper, one unit of zinc, and two units of glass. Each resistor requires three, three, and one unit of the three materials, respectively, and each computer chip requires two, one, and three units of these materials respectively. Putting this information into table form, we get:

<table>
<thead>
<tr>
<th>Component</th>
<th>Copper</th>
<th>Zinc</th>
<th>Glass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transistors</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Resistors</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Computer chips</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Supplies of these materials vary from week to week, so the company needs to determine a different production run each week. For example, one week the total amounts of materials available are 960 units of copper, 510 units of zinc, and 610 units of glass. Set up the system of equations modeling the production run and solve these equations for the number of transistors, resistors, and computer chips to be manufactured this week as follows:

a) By means of the linear algebra techniques of MATLAB,
b) By means of the symbolic capability of MATLAB,
c) Display the results from Part (b) by using the structure array syntax,
d) Display the results from Part (b) by specifying output names,
e) Add decimal points to the numbers in your equation definitions and solve them again. How do your answers change?
CIS 201 BUSINESS APPLICATIONS WITH MATLAB FALL 11-12 MIDTERM EXAM SOLUTIONS

1. First develop a hand solution to find $i$:

$$P = F(1 + i)^{-N}$$
$$\left(1 + i\right)^{N} = \frac{F}{P}$$
$$i = \left(\frac{F}{P}\right)^{\frac{1}{N}} - 1$$

Secondly develop a MATLAB solution by using symbolic mathematics as follows:
Define a symbolic equation and give it a name (recall that it is OK to put an equality inside the expression)

```matlab
>> X = sym('P = F*(1+i)^{-N}');
>> X = P = F*(1+i)^{-N}
% Now we can ask MATLAB to solve our equation. We need to specify that MATLAB is to solve for i,
% and i needs to be in single quotes, because it has not been separately defined as a symbolic variable:
>> solve(X, 'i')
ans =
(F/P) ^ (1/N)-1
```

2. The session is

```matlab
wage = [5, 5.5, 6.5, 6, 6.25]; hours = [40, 43, 37, 50, 45];
output = [1000, 1100, 1000, 1200, 1100];
earnings = wage.*hours
earnings =
200.0000   236.5000   240.5000   300.0000   281.2500
>> total_salary = sum(earnings)
total_salary =
1.2582e+003
>> total_widgets = sum(output)
total_widgets =
5400
>> average_cost = total_salary/total_widgets
average_cost =
0.2330
>> average_hours = sum(hours)/total_widgets
average_hours =
0.0398
>> [maximum, most_efficient] = max(output./earnings)
maximum =
5
most_efficient =
1
>> [minimum, least_efficient] = min(output./earnings)
minimum =
3.9111
least_efficient =
5
```
The workers earned $200, $236.50, $240.50, $300, and $281.25 respectively. The total salary paid out was $1258.20, and 5400 widgets were made. The average cost to produce one widget was 23.3 cents, and it took an average of 0.0398 hr to produce one widget. The first worker, who produced 5 widgets per dollar of earnings, was the most efficient. The fifth worker, who produced 3.911 widgets per dollar of earnings, was the least efficient.

3. Define the quantity of transistors, resistors, and computer chips as $x_1, x_2, x_3$. The system equations can then be defined as follows:

\[
\begin{align*}
4x_1 + 3x_2 + 2x_3 &= 960 \\
x_1 + 3x_2 + x_3 &= 510 \\
2x_1 + x_2 + 3x_3 &= 610
\end{align*}
\]

%% Solution of Problem 3
clear,clc
%% Part (a)
coef = [4 3 2; 1 3 1; 2 1 3];
result=[960; 510; 610];
answer=coef\result
%% Part (b)
a=sym('4*x1 + 3*x2 + 2*x3=960')
b=sym('x1 +3*x2 + x3=510')
c=sym('2*x1 +x2 + 3*x3=610')
solve(a,b,c)
%% Part (c)
answer=solve(a,b,c)
answer.x1
answer.x2
answer.x3

%% Part (d)
[x1,x2,x3]=solve(a,b,c)
%% Part (e)
a=sym('4.0*x1 + 3.0*x2 + 2.0*x3=960.0')
b=sym('1.0*x1 +3.0*x2 + 1.0*x3=510.0')
c=sym('2.0*x1 +1.0*x2 + 3.0*x3=610.0')

[x1,x2,x3]=solve(a,b,c)
% Without the decimal points the answers are expressed as fractions, with
% the decimal points they are expressed as floating point numbers