1. (20 p) Prove the following relationships among interest factors:
   a) \((P/F, i, N) = 1 – (P/A, i, N)i\)
   b) \((P/A, i, N \to \infty) = 1/i\)
   c) \(\left(\frac{P}{A}, r, N\right) = \left[\frac{e^{rN} - 1}{re^{rN}}\right]\)

2. (25 p)
   a) Determine the nominal interest rate \((r)\) when the effective annual interest rate \((i_a)\) is known to be 15% under the continuous compounding principle.
   b) What is the future worth of an equal quarterly payment series of $3500 for 15 years, if the interest rate is 11%, compounded weekly?
   c) At what rate of interest, compounded quarterly, will an investment triple itself in 10 years?

3. (25 p) The two cash flow transactions shown below are said to be equivalent at 6% interest compounded annually. Find the unknown value of \(X\) that satisfies the equivalence.

4. (30 p)
   a) What is equal end-of-month payments if you use an automobile loan of $20 000 at a nominal rate of 9% compounded monthly for 48 months.
   b) Complete the following table for the six payments, as you would expect a bank to calculate the values.

<table>
<thead>
<tr>
<th>Payment No.</th>
<th>Size of Payment ((A))</th>
<th>Principal Payment ((P_n))</th>
<th>Interest Payment ((I_n))</th>
<th>Loan Balance ((B_n))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
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<td>2</td>
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<td>3</td>
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<td>5</td>
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<tr>
<td>6</td>
<td></td>
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</tr>
</tbody>
</table>
1) 
   a) Formulas from Table 3.4 to be taken (Textbook):
   
   \( (P/A, i, N) = \left(1 + i\right)^{-N} \)
   \[ (P/A, i, N) = \frac{(1 + i)^N - 1}{i(1 + i)^N} \]
   
   \( (1 + i)^N \equiv 1 - \frac{(1 + i)^N - 1}{i(1 + i)^N} = \left(\frac{1}{1 + i}\right)^N \quad \forall N \)

   b) Formula from Table 3.4 to be taken (Textbook):
   
   \( (P/A, i, N \to \infty) \equiv \frac{1}{i} \)

   \[ \lim_{N \to \infty} \frac{(1 + i)^N - 1}{i(1 + i)^N} = \infty \quad \text{(L'Hôpital's rule will be applied to compute this limit)} \]

   \[ \lim_{N \to \infty} \frac{N(1 + i)^{N-1}}{iN(1 + i)^{N-1}} = \frac{1}{i} \]

   c) \( (P/A, r, N) = e^{rN} - 1 \)

   \[ P = \int_0^N Ae^{-rt} \, dt \quad f(t) = A \]

   present worth of cont. cash flow with cont. comp.

   \[ = \frac{1}{r} \left[ Ae^{-rt} \right]_0^N = \frac{A}{r} \left[ e^{-rN} - 1 \right] = A \left[ \frac{1}{r} \left( e^{-rN} - 1 \right) \right] = \frac{A}{r} \left[ e^{-rN} - 1 \right] = \frac{e^{rN} - 1}{re^{rN}} \]

   2) \( \text{(a)} \)

   Given: \( i_a = 0.15 \) (15%)

   Find: \( r \) (nominal interest rate)

   \( i_a = e^r - 1 \) (effective annual interest rate with cont. comp.)

   \[ 0.15 = e^r - 1 \rightarrow e^r = 1.15 \rightarrow r = \ln(1.15) = 0.1398(13.98\%) \]
Given:

\[ A = \$3500 \]
\[ K = 4; N = 4 \times 15 = 60 \]
\[ r = 11\%; M = 52; C = \frac{52}{4} = 13 \]

Find: \( F = ? \)

\[ F = A \left( F/A, i_p, N \right) \]
\[ i_p = \left( 1 + \frac{r}{M} \right)^C - 1 = \left( 1 + \frac{0.11}{52} \right)^{13} - 1 = 0.02785(2.785\%); \text{ effective interest rate per quarter} \]

From Table 3.4:

\[ \left( F/A, 2.785\%, 60 \right) = \left[ \frac{(1 + 0.02785)^6 - 1}{0.02785} \right] = 150,733 \]

\[ F = \$3500(150,733) = \$527565.50 \]

e)

Given:

\[ F = 3P; N = 10; M = 4 \]

Find: \( r \)

\[ F = P \left[ (1 + \frac{r}{M})^M \right]^N = P \left[ (1 + i_a)^M \right]^N = P \left( 1 + i_a \right)^N \]

\[ 3 = (1 + i_a)^{10} \rightarrow 1 + i_a = 3^{\frac{1}{10}} \rightarrow i_a = 3^{\frac{1}{10}} - 1 = 0.1161(11.61\%) \]

\[ i_a = \left( 1 + \frac{r}{M} \right)^M - 1 \rightarrow 0.1161 = \left( 1 + \frac{r}{4} \right)^4 - 1 \rightarrow \left( 1 + \frac{r}{4} \right)^4 = 1.1161 \rightarrow r = 4 \left[ (1,1161)^{\frac{1}{4}} - 1 \right] \]

\[ r = 0.1114(11.14\%) \]

3)
Given:

\( CFI \equiv CF2 \)

Find: \( X \)

Select reference point at \( n = 0 \)

\[
(P_0)_{CF1} = $400 \left( \frac{P}{A}, 6\%, 5 \right) - $100 \left( \frac{P}{F}, 6\%, 4 \right) = $400(4,2124) - $100(0,7921), Table / p.881
\]

\[
(P_0)_{CF1} = $1605,75
\]

\[
(P_0)_{CF2} = X \left( \frac{P}{A}, 6\%, 5 \right) - (400 + X) \left( \frac{P}{F}, 6\%, 4 \right) = X(4,2124) - (400 + X)(0,7921), Table / p.881
\]

\[
$1605,75 = 3,4203X - 316,84 \rightarrow X = $562,11
\]

2nd Way:

\[
(P_0)_{CF1} = $400 \left( \frac{P}{A}, 6\%, 3 \right) + $300 \left( \frac{P}{F}, 6\%, 4 \right) + $400 \left( \frac{P}{F}, 6\%, 5 \right) =
\]

\[
(P_0)_{CF1} = 400(2,6730) + 300(0,7921) + 400(0,7473)
\]

\[
(P_0)_{CF1} = $1605,75
\]

\[
(P_0)_{CF2} = X \left( \frac{P}{A}, 6\%, 3 \right) - $400 \left( \frac{P}{F}, 6\%, 4 \right) + X \left( \frac{P}{F}, 6\%, 5 \right)
\]

\[
(P_0)_{CF2} = X(2,6730) - $400(0,7921) + X(0,7473)
\]

\[
(P_0)_{CF2} = 3,4203X - 316,84
\]

\[
$1605,75 = 3,4203X - 316,84 \rightarrow X = $562,11
\]

4)(a)

Given:

\( P = $20000; r = 9\%; M = K = 12 \)

\( N = 48 \)

Find: \( A \)

\[
A = P \left( \frac{A}{P}, i_c, N \right) = i_p = \frac{r}{M} = 0.09 = 0.0075(0,75\% \text{ effective interest rate per month/payment})
\]

\[
A = $20000 \left( \frac{A}{P}, 0,75\%, 48 \right) = $20000(0,0249); Table / p.872
\]

\( A = $498 \)

b) \( P_n, I_n, B_n \) will be calculated according to the tabular method as follows:

\[
P_n = A - I_n, I_n = (B_{n,i})(i); B_n = B_{n-1} - P_n
\]

<table>
<thead>
<tr>
<th>Payment No.</th>
<th>Size of Payment ((A, S))</th>
<th>Principal Payment ((P_n, S))</th>
<th>Interest Payment ((I_n, S))</th>
<th>Loan Balance ((B_n, S))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>498</td>
<td>498-150=348</td>
<td>20000(0.0075)=150</td>
<td>20000-348=19652</td>
</tr>
<tr>
<td>2</td>
<td>498</td>
<td>498-147,39=350,61</td>
<td>19652(0.0075)=147,39</td>
<td>19652-350,61=19301,39</td>
</tr>
<tr>
<td>3</td>
<td>498</td>
<td>498-144,76=353,24</td>
<td>19301,39(0.0075)=144,76</td>
<td>19301,39-353,24=18948,15</td>
</tr>
<tr>
<td>4</td>
<td>498</td>
<td>498-142,11=355,89</td>
<td>18948,15(0.0075)=142,11</td>
<td>18948,15-355,89=18592,26</td>
</tr>
<tr>
<td>5</td>
<td>498</td>
<td>498-139,44=358,55</td>
<td>18592,26(0.0075)=139,44</td>
<td>18592,26-358,55=18233,70</td>
</tr>
<tr>
<td>6</td>
<td>498</td>
<td>498-136,75=361,25</td>
<td>18233,70(0.0075)=136,75</td>
<td>18233,70-361,25=17872,45</td>
</tr>
</tbody>
</table>