1. (20 p) Prove the following relationships among interest factors:
   a) \( (A/P, i, N) = \frac{i}{1 - (P/F, i, N)} \)
   b) \( (A/P, i, N \to \infty) = i \)
   c) \( \left( \frac{F}{A}, r, N \right) = \left[ \frac{e^{rN} - 1}{r} \right] \)

2. (25 p)
   a) What is the future worth of an equal payment series of $3000 each quarter for five years if the interest rate is 8% compounded continuously?
   b) If your credit card calculates interest based on 12.5% APR, what are your monthly interest rate and annual effective interest rate?
   c) At what rate of interest, compounded daily, will an investment double itself in 5 years?

3. (25 p) What value of \( F_3 \) would be equivalent to the payments shown in the cash flow diagram below? Assume that the interest rate is 10%, compounded annually.

4. (30 p) Suppose you borrowed $10,000 at an interest rate of 12%, compounded monthly over 36 months. At the end of the first year (after 12 payments), you want to negotiate with the bank to pay off the remainder of the loan in 8 equal quarterly payments. What is the amount of this quarterly payment, if the interest rate and compounding frequency remain the same?
1) a) Formulas from Table 3.4 to be taken (Textbook):
\[
\left( \frac{P}{F}, i, N \right) = (1 + i)^{-N}
\]
\[
\left( \frac{A}{P}, i, N \right) = \frac{i(1 + i)^N}{(1 + i)^N - 1}
\]
\[
\frac{i(1 + i)^N}{(1 + i)^N - 1} \Rightarrow \frac{i}{1 - (1 + i)^{-N}} = \frac{i}{1 - \frac{1}{(1 + i)^N}} = \frac{i(1 + i)^N}{(1 + i)^N - 1}
\]

b) Formula from Table 3.4 to be taken (Textbook):
\[
\left( \frac{A}{P}, i, N \rightarrow \infty \right) = i
\]
\[
\lim_{N \to \infty} \frac{i(1 + i)^N}{(1 + i)^N - 1} = \frac{\infty}{\infty} (L'Hôpital's rule will be applied to compute this limit)
\]
\[
\lim_{N \to \infty} \frac{i(1 + i)^N}{(1 + i)^N - 1} = \left[ \frac{iN(1 + i)^{N-1}}{N(1 + i)^{N-1}} \right] = i
\]

c) \[
\left( \frac{F}{A}, r, N \right) = e^{rN} - 1
\]
\[
F = \int_0^N Ae^r dt ; f(t) = A
\]
future worth of cont.cash flow with cont.comp.
\[
F = \frac{1}{r} \int_0^N Ae^r (r dt) = \frac{A}{r} e^{rN} \bigg|_0^N = \frac{A}{r} \left[ e^{rN} - 1 \right] = A \left[ e^{rN} - 1 \right] \to \left( \frac{F}{A}, r, N \right) = \frac{e^{rN} - 1}{r}
\]

2) (a) 
Given :
\[
A = \$3000
\]
\[
K = 4; N = 4 \times 5 = 20
\]
\[
r = 8\%; M = \infty; C = \infty
\]
Find : \( F = ? \)
\[
F = A \left( \frac{F}{A}, i_p, N \right)
\]
\[
i_p = e^{i/4} - 1 = e^{0.08/4} - 1 = 0.0202 (2.02\%); \text{effective interest rate per quarter}
\]
From Table 3.4 :
\[
\left( \frac{F}{A}, 2.02\%, 20 \right) = \left[ \frac{(1 + 0.0202)^{20} - 1}{0.0202} \right] = 24,3458
\]
\[
F = \$3000 \left( 24,3458 \right) = \$73037.45
\]
(b)

Given: \( r = 12.5 \% \text{ APR}; M = 12 \)
Find: \( i_a \) (effective annual interest rate)

\[
i_a = \left(1 + \frac{r}{M}\right)^M - 1 = \left(1 + \frac{0.125}{12}\right)^{12} - 1 = 0.1324 \approx (13.24\%)
\]

c)  

Given:  
\( F = 2P; N = 5; M = 365 \)
Find : \( r \)

\[
F = P \left[ \left(1 + \frac{r}{M}\right)^M \right] = P \left[ \left(1 + i_e \right)^w \right] = P \left(1 + i_e \right)^N
\]

\[
2 = \left(1 + i_a \right)^5 \rightarrow 1 + i_a = 2^{\frac{1}{5}} \rightarrow i_a = 2^{\frac{1}{5}} - 1 = 0.1487 \approx (14.87\%)
\]

\[
i_a = \left(1 + \frac{r}{M}\right)^M - 1 \rightarrow 0.1487 = \left(1 + \frac{r}{365}\right)^{365} - 1 \rightarrow \left(1 + \frac{r}{365}\right)^{365} = 1.1487 \rightarrow r = 365 \left(1.1487 \right)^{\frac{1}{365}} - 1
\]

\[
r = 0.1387 \approx (13.87\%)
\]

3)

Given:  
\( CF1 \equiv F_3 \)
Find : \( F_3 \)

Select reference point at \( n = 3 \)

\[
(V_3)_{CF1} = \$100 \left(F/P,10\%,3\right) + \$100 \left(F/A,10\%,2\right) \left(F/P,10\%,1\right)
\]

\[
+ \$50 \left(P/A,10\%,3\right) = \$100(1,3310) + \$100(2,1000)(1,1000) + \$50(2,4869) \text{ Table / p.885}
\]

\[
(V_3)_{CF1} = \$488.45
\]

\[
\rightarrow F_3 = \$488.45
\]

2ndWay :

\[
(V_3)_{CF1} = \$100 \left(F/P,10\%,3\right) + \$100 \left(F/P,10\%,2\right) + \$100 \left(F/P,10\%,1\right)
\]

\[
+ \$50 \left(P/F,10\%,1\right) + \$100 \left(P/F,10\%,2\right) + \$100 \left(P/F,10\%,3\right)
\]

\[
(V_3)_{CF1} = \$100(1,3310) + \$100(1,2100) + \$100(1,1000)
\]

\[
+ \$50(0.9091) + \$50(0.8264) + \$50(0.7513)
\]

\[
\rightarrow F_3 = \$488.44
\]
4)(a)  
Given:  
\[ P = $10000; r = 12\%; M = K = 12 \]  
\[ N = 12 \times 3 = 36 \]  
Find: \( A \)  
\[ A = P \left( \frac{A}{P}, i_c, N \right) \]  
\( i_c = i_p = \frac{r}{M} = \frac{0.12}{12} = 0.01(1\% \text{ effective interest rate per month/payment}) \)  
\[ A = $10000 \left( \frac{A}{P}, 1\%, 36 \right) = $10000(0.0332); \text{Table / p.873} \]  
\( A = $332 \)

b) \( B_n \) remainder of loan at the end of 1st year after 12 payments will be calculated according to the remaining balance method as follows:  
\[ B_n = A \left( \frac{A}{P}, 1\%, N-n \right); N=36, n=12 \]  
\( B_{12} = $332 \left( \frac{A}{P}, 1\%, 24 \right) = $332(21,2434) = $7052,81(\text{Table/p.873}) \) will be paid off in 8 equal quarterly payments  
\( p_i = $7052,81; r=12\%; K=4; N_i=4 \times 2=8; M=12 \)  
\[ i_p = \left( 1 + \frac{r}{M} \right)^K - 1 = \left( 1 + \frac{0.12}{12} \right)^4 - 1 = 0.0406(4.06\%) \text{ effective interest rate per quarter} \]  
\[ A = P_i \left( \frac{A}{P}, i_p, N_i \right) = $7052,81 \left[ \frac{0.0406(1 + 0.0406)^8}{(1 + 0.0406)^8 - 1} \right] = $1050,05 \]