1. (35 p) Suppose you deposit $1,000 at the end of each quarter for 5 years at an interest of 8% compounded continuously. What equal end-of-year deposit over 5 years would accumulate the same amount at the end of 5 years under the same interest compounding (8%, compounded continuously)?

```
Quarters: 0  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20

Years:

$1,000

0  1  2  3  4  5
A  A  A  A  A  A
```

2. (30 p) If you make the following series of deposits at an interest rate of 12%, compounded weekly, what would be the total balance at the end of 10 years?

<table>
<thead>
<tr>
<th>End of Period</th>
<th>Amount of Deposit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$800</td>
</tr>
<tr>
<td>1–9</td>
<td>$1500</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

```

0  1  2  3  4  5  6  7  8  9 10

$800

$1500
```

3. (35 p) You are considering purchasing a piece of industrial equipment that costs $30,000. You decide to make a down payment in the amount of $5,000 and to borrow the remainder from a local bank at an interest rate of 9%, compounded monthly. The loan is to be paid off in 36 monthly installments.
   a) What is the amount of the monthly payment (A)?
   b) What is your interest payment and principal payment for the period six (6th month) as you would expect a bank to calculate the values?
   c) You have made 24 payments and want to figure out the balance remaining immediately after 24th payment. What is that balance?
1. Given: deposit frequency=$1000 per quarter  
   \( r = 8\% \) compounded continuously  
   \( N = 20 \) quarters  
   Find: Equivalent annual deposit amount \((A)\)  

   \( i_p \) should be determined according to the payment period \((K=4)\) of the first cash flow by considering  
   compounding continuously \((C \rightarrow \infty)\) with the following equation:

   \[
   i_p = e^{\frac{r}{K}} - 1 = e^{0.08/4} - 1 = 0.02020(2.02\%)
   \]

   Second cash flow is based on equal end-of-year deposit over 5 years \((K=1, C = M \rightarrow \infty M)\) with the same interest compounding. \(i_a\) effective annual interest should be determined as follows:

   \[
   i_a = e^r - 1 = e^{0.08} - 1 = 0.08329(8.33\%)
   \]

   \(A\) can be calculated by equating the present worth of the first cash flow to the present worth of the second cash flow:

   \[
   $1000\left(\frac{P}{A}, 2.02\%, 20\right) = A\left(\frac{P}{A}, 8.33\%, 5\right)
   \]

   From Table 3.4 (Textbook/Park 4th ed.):

   \[
   $1000 \left[ \frac{(1 + 0.0202)^{20} - 1}{0.0202(1 + 0.0202)^{20}} \right] = A \left[ \frac{(1 + 0.0833)^5 - 1}{0.0833(1 + 0.0833)^5} \right]
   \]

   \(A = \$4122.93\)

2. Given: \(A_0 = \$800, A = \$1500, r = 12\% \) compounded weekly \((K=1, M=CK=52), N=10 \) years  
   Find: \(F\)  

   First effective interest rate per payment period \((i_p)\) should be determined from the following equation:

   \[
   i_p = \left[ 1 + \frac{r}{CK} \right]^C - 1 = \left[ 1 + \frac{0.12}{(52)(1)} \right]^{52} - 1 = 0.1273(12.73\%)
   \]

   Note that there are two cash flow components in the series. The first one is a single payment amount \(\$800) at the period 0 and the other is the \$1500 equal payment series. Also we are looking for an equivalent value of these payments at the end of year 10, not year 9. Therefore, we may solve the problem in two steps. First find the equivalent future worth of the single payment at the period of 10. Then find the equivalent future worth amount of the \$1500 payment series at the end of year 10.

   **Single-payment:**

   \[
   V_1 = \$800\left(\frac{F}{P}, 12.73\%, 10\right) = \$800\left(1 + 0.1273\right)^{10} = \$800(3.3143) = \$2651.44(\text{Formula : Table3.4})
   \]

   **Equal-payment series:** First find the equivalent future worth of the series at the end of year 9 and multiply this amount by \((F/P, 12.73\%, 1)\) to obtain the value at year 10.

   \[
   V_2 = \$1500\left(\frac{F}{A}, 12.73\%, 9\right)\left(\frac{F}{P}, 12.73\%, 1\right) = \$1500 \left[ \frac{(1 + 0.1273)^9 - 1}{0.1273} \right] \left(1 + 0.1273\right)
   \]

   \[
   = \$1500(15.2400)(1.1273) = \$25770.13
   \]
Total:
\[ F = V_1 + V_2 = 2651.44 + 25770.13 = 28421.57 \]

3. a) 
Given:
\[ P = 30000 - 5000 = 25000; r = 9\%; C = 1, M = K = 12 \]
\[ N = 36 \]
Find : \( A \)
\[ A = P \left( A / P, i_p, N \right) \]
\[ i_p = \frac{r}{M} = \frac{0.09}{12} = 0.0075(0.75\% \text{ effective interest rate per month/payment}) \]
\[ A = 25000 \left( A / P, 0.75\%, 36 \right) = 25000(0.0318); Table / p.872 \]
\[ A = 795 \]

b) For the period 6 (\( n=6 \)) Principal payment (\( P_n \)), interest payment (\( I_n \)) and balance at the end of the period (\( B_n \)) will be calculated according to the tabular method as follows:
\[ B_n = A \left( P / A, 0.75\%, N - n \right) I_n = (B_{n-1}); P_n = A - I_n \]
\[ B_{n-1} = B_5 = A \left( P / A, 0.75\%, 36 - 5 \right) = 795 \left( P / A, 0.75\%, 31 \right) = 795(27.5683) = 21916.80 \]
\[ (\text{Factor taken from Table/p.872}) \]
\[ I_6 = B_5i = (21916.80)(0.0075) = 164.38 \]
\[ P_6 = A - I_6 = 795 - 164.38 = 630.62 \]

c) 
\[ B_{24} = A \left( P / A, 0.75\%, 36 - 24 \right) = A \left( P / A, 0.75\%, 12 \right) = (795(11.4349)) = 9090.75 \]
\[ \text{Factor taken from Table/p.872} \]