1. (35 p) Suppose you deposit $1,000 at the end of each quarter for 5 years at an interest of 9% compounded semiannually. What equal end-of-year deposit over 5 years would accumulate the same amount at the end of 5 years under the same interest compounding (9%, compounded semiannually)?

![Diagram showing quarterly deposits](image)

2. (35 p) For the quarterly cash flow transactions shown below, determine the amount of money in the account at the end of year 4 if the rate is 10% compounded weekly. Assume that deposits (+) and withdrawal(-) cash flows start to earn/charge interest immediately.

<table>
<thead>
<tr>
<th>End of Quarter</th>
<th>Amount of Deposit (+), $/Quarter</th>
<th>Amount of Withdrawal (-), $/Quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>1200</td>
<td></td>
</tr>
<tr>
<td>6-10</td>
<td>800</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>1000</td>
<td>3000</td>
</tr>
<tr>
<td>12</td>
<td>800</td>
<td>500</td>
</tr>
<tr>
<td>13</td>
<td>600</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>-</td>
<td>2000</td>
</tr>
</tbody>
</table>

3. (30 p) You are considering buying a home for $250,000. If you make a down payment of $100,000 and take out a mortgage on the rest of the money at 12% compounded monthly, what will be your monthly payment to retire the mortgage in 20 years?
1. Given: deposit frequency=$1000 per quarter  
   \[ r = 9\% \text{ compounded semiannually} \]  
   \[ N = 20 \text{ quarters} \]

   Find: Equivalent annual deposit amount \((A)\) of the second cash flow for the same \(F\) value with the 1st cash flow.

   **First cash flow:**
   \(i_p\) should be determined according to the payment period of the first cash flow by considering compounding semiannually:

   \[
   K = 4, M = 2  
   C = \frac{M}{K} = \frac{2}{4} = 0.5  
   \]

   \[
   i_c = \frac{r}{M} = \frac{0.09}{2} = 0.045  
   \]

   \[
   i_p = (1 + i_c)^C - 1 = (1 + 0.045)^{0.5} - 1 = 0.0223 (2.23\%) \]

   **Second cash flow:**
   It is based on equal end-of-year deposit over 5 years with the same interest compounding.

   \[
   K = 1, M = 2  
   C = \frac{M}{K} = \frac{2}{1} = 2  
   \]

   \[
   i_c = \frac{r}{M} = \frac{0.09}{2} = 0.045  
   \]

   \[
   i_p = (1 + i_c)^C - 1 = (1 + 0.045)^2 - 1 = 0.0920 (9.20\%)  
   \]

   Effective annual interest \(i_a\) = \(i_p\) = 0.0920 (9.20%)

   \(A\) can be calculated by equating the future worth of the first cash flow to the future worth of the second cash flow as follows:

   \[
   \$1000(F/A, 2.23\%, 20) = A (F/A, 9.20\%, 5)  
   \]

   From the equal payment series compound amount formula (Table 3.4/Textbook/Park 4th ed.):

   \[
   \$1000 \left[ \frac{(1 + 0.0223)^{20} - 1}{0.0223} \right] = A \left[ \frac{(1 + 0.0920)^5 - 1}{0.0920} \right]  
   \]

   \[
   A = $4137.68  
   \]

2. Given: quarterly cash flow on the table with nominal interest rate 10% compounded weekly.

   Find: \(F\)

   \[
   K = 4, M = 52, N = 16 \text{ quarters}  
   C = \frac{M}{K} = \frac{52}{4} = 13  
   \]

   \[
   i_c = \frac{r}{M} = \frac{0.10}{52}  
   \]

   First \(i_p\) should be determined according to the quarterly payment period of the cash flow by considering compounding weekly:

   \[
   i_p = (1 + i_c)^C - 1 = (1 + \frac{0.10}{52})^{13} - 1 = 0.0253 (2.53\%)  
   \]

   Convention in terms of the account: Deposits (+), Withdrawals(-)

   Note that there are quarterly cash flows (as deposits and withdrawals) given in the table.
1st Cash Flow: Equal deposit (Uniform $1200) series (Quarter:1-5),
2nd Cash Flow: Equal deposit (Uniform $800) series (Quarter:6-10),
3rd Cash Flow: Decreasing arithmetic gradient (Uniform part:$1000, decreasing gradient part, G=$200) deposit series (Quarters:11-15)
4th Cash Flow: Single withdrawal amount of $3000 (Quarter:11),
5th Cash Flow: Single withdrawal amount of $500 (Quarter:12),
6th Cash Flow: Single withdrawal amount of $2000 (Quarter:16),

Equivalent value of these deposits and withdrawals cash flows at the end of year 4 (end of quarter period 16) can be found as follows:

\[ F_1 = A_1 (F/A, 2.53\%, 5)(F/P, 2.53\%, 11) = 1200 \frac{(1 + 0.0253)^5 - 1}{0.0253} (1 + 0.0253)^{11} \]

\[ F_2 = A_2 (F/A, 2.53\%, 5)(F/P, 2.53\%, 6) = 800 \frac{(1 + 0.0253)^5 - 1}{0.0253} (1 + 0.0253)^6 \]

\[ F_3 = \left[ 1000 - 200 \left( \frac{(1 + 0.0253)^5 - (0.0253)(5) - 1}{0.0253} \right) \right] \frac{(1 + 0.0253)^5 - 1}{0.0253} (1 + 0.0253)^1 \]

\[ F_4 = P_4 (F/P, 2.53\%, 5) = -3000(1 + 0.0253)^5 \]

\[ F_5 = P_5 (F/P, 2.53\%, 4) = -500(1 + 0.0253)^4 \]

\[ F_6 = P_6 (F/P, 2.53\%, 0) = -2000(1 + 0.0253)^0 = -2000 \]

\[ F = F_1 + F_2 + F_3 + F_4 + F_5 + F_6 = 10533.52 \]

Factors are taken from the Table 3.4 of the formulas

3. Given:

\[ P = 2500000 - 1000000 = 150000 \]
\[ r = 12\%, M = 12 \text{ (compounding monthly)}, K = 12 \text{ (monthly payment)} \]
\[ C = \frac{M}{K} = \frac{12}{12} = 1 \]
\[ i_p = (1 + i_c)^C - 1 = i_c = \frac{r}{M} = 0.12 = 0.01 \text{ (1\%)} \]
\[ N = Mn = 12(20) = 240 \text{ months} \]

Find: \( A \)

\[ A = P(A/P, 1\%, 240) = 150000(0.0110) = 1651.63 \]

From Table/p.873