1. (35 p) Suppose you deposit $1,000 at the end of each quarter for 5 years at an interest of 6% compounded continuously. What equal end-of-year deposit over 5 years would accumulate the same amount at the end of 5 years under the same interest compounding (6%, compounded continuously)?

2. (35 p) For the semiannual cash flow transactions shown below, determine the amount of money in the account at the end of year 5 if the rate is 8% compounded monthly. Assume that deposits (+) and withdrawal(-) cash flows start to earn/charge interest immediately.

<table>
<thead>
<tr>
<th>End of Semiannual Period</th>
<th>Amount of Deposit (+), $/Period</th>
<th>Amount of Withdrawal (-), $/Quarter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-4</td>
<td>1500</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2000</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2250</td>
<td>2000</td>
</tr>
<tr>
<td>7</td>
<td>2500</td>
<td>2000</td>
</tr>
<tr>
<td>8</td>
<td>2750</td>
<td>2000</td>
</tr>
<tr>
<td>9</td>
<td>3000</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

3. (30 p) Suppose you borrowed $20,000 at an interest rate of 9%, compounded monthly over 24 months. At the end of the first year (after 12 payments), you want to negotiate with the bank to pay off the remainder of the loan in 4 equal quarterly payments. What is the amount of this quarterly payment, if the interest rate and compounding frequency remain the same?
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SOLUTIONS

1. Given: deposit frequency=$1000 per quarter
   \[ r = 9\% \text{ compounded continuously} \]
   \[ N = 20 \text{ quarters} \]

Find: Equivalent annual deposit amount \((A)\) of the second cash flow for the same \(F\) value with the 1st cash flow.

**First cash flow:**
\(i_p\) should be determined according to the payment period of the first cash flow by considering compounding continuously:

\[
K = 4, M = \infty \\
C = \infty \\
\]
\[
i_p = e^{r/K} - 1 = e^{0.06/4} - 1 = 0.0151 (1.51\%) \\
\]

**Second cash flow:**
It is based on equal end-of-year deposit over 5 years with the same interest compounding.

\[
K = 1, M = \infty \\
C = \infty \\
\]
\[
i_p = i_a = e^{r/K} - 1 = e^{0.06/1} - 1 = 0.0618 (6.18\%) \\
Effective annual interest = i_a = e^r - 1 = 0.0618 (6.18\%) \\
\]
\(A\) can be calculated by equating the future worth of the first cash flow to the future worth of the second cash flow as follows:

\[
$1000(F/A, 1.51\%, 20) = A (F/A, 6.18\%, 5) \\
\]
From the equal payment series compound amount formula (Table 3.4/Textbook/Park 4th ed.):

\[
$1000 \left[ \frac{(1 + 0.0151)^{20} - 1}{0.0151} \right] = A \left[ \frac{(1 + 0.0618)^5 - 1}{0.0618} \right] \\
A = \$4091.37 \\
\]

2. Given: semiannually cash flow on the table with nominal interest rate 8% compounded monthly.
Find: \(F\)

\[
K = 2, M = 12, N = 10 \text{ periods} \\
C = \frac{M}{K} = \frac{12}{2} = 6 \\
\]
\[
i_c = \frac{r}{M} = \frac{0.08}{12} \\
\]
First \(i_p\) should be determined according to the semiannually payment period of the cash flow by considering compounding monthly:

\[
i_p = (1 + i_c)^C - 1 = (1 + \frac{0.08}{12})^6 - 1 = 0.0407 (4.07\% \text{ per period}) \\
\]

**Convention in terms of the account:** Deposits (+), Withdrawals(-)

Note that there are semiannually cash flows (as deposits and withdrawals) given in the table.
1st Cash Flow: Equal deposit (Uniform $1500) series (Period:1-4),
2nd Cash Flow: Increasing arithmetic gradient (Uniform part:$2000, \(G=$250) deposit series (Period:5-9),
3rd Cash Flow: Equal withdrawal (Uniform $2000) series (Period:6-8),
Equivalent value of these deposits and withdrawals cash flows at the end of year 5 (end of semiannual period 10) can be found as follows:

\[ F_1 = A_1(F/A, 4.07\%, 4)(F/P, 4.07\%, 6) = 1500 \left( \frac{(1 + 0.0407)^4 - 1}{0.0407} \right) (1 + 0.0407)^6 = 8100.74 \]

\[ F_2 = \left[ A_2 + G(A/G, 4.07\%, 5)(F/A, 4.07\%, 5)(F/P, 4.07\%, 1) \right] \left[ \frac{1 + (1 + 0.0407)^5 - (0.0407)(5) - 1}{0.0407[(1 + 0.0407)^5 - 1]} \right] \left[ \frac{(1 + 0.0407)^5 - 1}{0.0407} \right] (1 + 0.0407)^1 \]

\[ F_2 = 13999.13 \]

\[ F_3 = A_3(F/A, 4.07\%, 3)(F/P, 4.07\%, 2) = -$2000 \left[ \frac{(1 + 0.0407)^3 - 1}{0.0407} \right] (1 + 0.0407)^2 \]

\[ F_3 = -$6766.41 \]

\[ F = F_1 + F_2 + F_3 = 8100.74 + 13999.13 - -$6766.41 = $15333.46 \]

Factors are taken from the Table 3.4 of the formulas

3. Given:

\[ P = $20000, n = 2 \text{ years}, N = Mn = 12(2) = 24 \text{ months} \]

\[ r = 9\%, M = 12 \text{ (compounding monthly)}, K = 12 \text{ (monthly payment)} \]

\[ C = \frac{M}{K} = \frac{12}{12} = 1 \]

\[ i_c = (1 + i_c)^c - 1 = i_c = \frac{r}{M} = \frac{0.09}{12} = 0.0075 \text{ (0.75\%)} \]

Find: \( A \)

\[ A = P(A/P, 0.75\%, 24) = $20000(0.0457) = $914.00 \]

From Table/p.872

Remainder of loan at the end of 1st year after 12 monthly payments will be calculated according to the remaining balance method as follows:

\[ B_{12} = A(P/A, 0.75\%, 24 - 12) = $914.00(11.4349) = $10451.50 \]

From Table/p.872

This remaining balance ($10451.50) will be paid off in 4 equal quarterly payments.

\[ K = 4, M = 12, N = 4 \text{ periods} \]

\[ C = \frac{M}{K} = \frac{12}{4} = 3 \]

\[ i_c = \frac{r}{M} = \frac{0.09}{12} \]

First \( i_p \) should be determined according to the quarterly payment period of the cash flow by considering compounding monthly:

\[ i_p = (1 + i_c)^c - 1 = (1 + \frac{0.09}{12})^3 - 1 = 0.0227 \text{ (2.27\% per quarter)} \]

\[ A_Q = P(A/P, 2.27\%, 4) = $10451.50 \left[ \frac{0.0227(1 + 0.0227)^4}{(1 + 0.0227)^4 - 1} \right] = $2762.82 \]

Factor \( (A/P, i, N) \) was taken from the Table 3.4 of the formulas

ECON305_Spring11-12MTMakeupQsProf.Dr. Hüseyin Oğuz