1. (10p) You have been offered a credit card by a bank that charges interest at 21.6%, compounded monthly.
   a) What is the effective interest rate per month for this credit card?
   b) What is the effective annual interest rate for this credit card?

2. (10p) What is the future worth of an equal quarterly payment series of $2,500 for 10 years, if the interest rate is 9%, compounded quarterly?

3. (30p) What value of $F_3$ would be equivalent to the payments shown in the cash flow diagram below? Assume that the interest rate is 10%, compounded annually.

4. (30p) Suppose you borrowed $10,000 at an interest rate of 12%, compounded monthly over 36 months. At the end of the first year (after 12 payments), you want to negotiate with the bank to pay off the remainder of the loan in 8 equal quarterly payments. What is the amount of this quarterly payment, if the interest rate and compounding frequency remain the same?

5. (20p) A business school has just completed a new building complex worth $50 million. A campaign targeting alumni is planned to raise funds for future maintenance costs, which are estimated at $2 million per year. Any unforeseen costs above $2 million per year would be obtained by raising tuition. Assuming that the school can create a trust fund that earns 8% interest annually, how much has to be raised now to cover the perpetual string of $2 million annual costs?
1. Given: \( r = 21.6\% \) compounded monthly

Find: a) effective interest rate per month (\( i \))

\[
i = \frac{r}{M} = \frac{0.216}{12} = 0.018(1.8\%)
\]

b) effective annual interest rate (\( i_a \))

\[
i_a = \left(1 + \frac{r}{M}\right)^M - 1 = (1 + i)^M - 1 = (1 + 0.018)^{12} - 1 = 0.2387(23.87\%)
\]

2. Given: \( A = $2500 \), \( r = 9\% \) compounded quarterly, and \( N = 10 \times 4 = 40 \) quarters

Find: \( F \)

Compounding period (quarterly) is the same as the payment period (quarterly).

Effective interest rate per quarter (\( i \)):

\[
i = \frac{r}{M} = \frac{0.09}{4} = 0.0225(2.25\%) per quarter
\]

\[
F = A\left(\frac{F}{A},i,N\right) = $2500\left(\frac{F}{A},0.25\%,40\right) = $2500\left[\frac{(1+i)^N - 1}{i}\right] = $2500\left[\frac{(1 + 0.0225)^{40} - 1}{0.0225}\right] = $159465.44
\]

\( (F/A \) Factor taken from Textbook/Table 3.4) 

3. 

Given:

\( CF_1 \equiv F_3 \)

Find: \( F_3 \)

Select reference point at \( n = 3 \)

\[
(V_3)_{CF_1} = $100\left(\frac{F}{P},10\%,3\right) + $100\left(\frac{F}{A},10\%,2\right)\left(\frac{F}{P},10\%,1\right)
\]

\[
+ $50(\frac{P}{A},10\%,3) = $100(1,3310) + $100(2,1000)(1,1000) + $50(2,4869) Table / p.885
\]

\[
(V_3)_{CF_1} = $488,45
\]

\( \rightarrow F_3 = $488,45 \)

2nd Way:

\[
(V_3)_{CF_1} = $100\left(\frac{F}{P},10\%,3\right) + $100\left(\frac{F}{P},10\%,2\right) + $100\left(\frac{F}{P},10\%,1\right)
\]

\[
+ $50\left(\frac{P}{F},10\%,1\right) + $50\left(\frac{P}{F},10\%,2\right) + $50\left(\frac{P}{F},10\%,3\right)
\]

\[
(V_3)_{CF_1} = $100(1,3310) + $100(1,2100) + $100(1,1000)
\]

\[
+ $50(0,9091) + $50(0,8264) + $50(0,7513)
\]

\( \rightarrow F_3 = $488,44 \)
4. 

Given:

\( P = $10000; r = 12\%; M = K = 12 \)

\( N = 12 \times 3 = 36 \)

Find: \( A \)

\[
A = P \left( \frac{A}{P, i_c, N} \right) i_c = \frac{r}{M} = \frac{0.12}{12} = 0.01(1\% \text{ effective interest rate per month/payment})
\]

\( A = $10000 \left( \frac{A}{P, 1\%, 36} \right) = $10000(0.0332); \text{Table/p.873} \)

\( A = $332 \)

\( B_n \) remainder of loan at the end of 1\(^{st}\) year after 12 payments will be calculated according to the remaining balance method as follows:

\[ B_n = A \left( \frac{P}{A, 1\%, N-n} \right); N=36, n=12 \]

\( B_{12} = $332 \left( \frac{P}{A, 1\%, 24} \right) = $332(21.2434) = $7052.81 (\text{Table/p.873}) \) will be paid off in 8 equal quarterly payments

\[ P_t = $7052.81; r = 12\%; K = 4; N_1 = 4 \times 2 = 8; M = 12 \]

\( i_p = \left( 1 + \frac{r}{M} \right)^{M/K} - 1 = \left( 1 + \frac{0.12}{12} \right)^3 - 1 = 0.0303(3.03\% \text{ effective interest rate per quarter}) \)

\[ A = P_t \left( \frac{A}{P, i_p, N_1} \right) = $7052.81 \left[ \frac{0.0303(1 + 0.0303)^3}{(1 + 0.0303)^3 - 1} \right] = $1005.99 \]


Given: \( A = $2000000, r = i = 8\% \text{ per year, and } N = \infty \text{ perpetual cash flow} \)

Find: \( CE \)

The capitalized equivalent (\( CE \)) cost of perpetual cash flow:

\[ CE(i) = \frac{A}{i} = \frac{$2000000}{0.08} = $25000000 \]

Comments: It is easy to see that this lump-sum amount should be sufficient to pay maintenance expenses for the school forever. Suppose the school deposited $25 million at a bank that paid 8\% interest annually. At the end of the first year, the $25 million would earn:

\( I = Pi = ($25000000)(0.08) = $2000000 \text{ interest} \)

If this interest \( I \) were withdrawn, the $25 million would remain in the account. At the end of second year, the $25 million balance would again earn

\( I = Pi = ($25000000)(0.08) = $2000000 \text{ interest} \)

The annual withdrawal could be continued forever, and the endowment (gift funds) would always remain at $25 million.