The annual cash flows (in thousands) associated with a company are shown below at a nominal interest rate of 15% compounded quarterly:

<table>
<thead>
<tr>
<th>Years</th>
<th>Annual Cash Flow, $/year</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>1000</td>
</tr>
<tr>
<td>4-6</td>
<td>2000</td>
</tr>
<tr>
<td>7-9</td>
<td>3000</td>
</tr>
<tr>
<td>10-12</td>
<td>4000</td>
</tr>
</tbody>
</table>

(a) What equal end-of-quarter deposit over twelve (12) years would accumulate the same amount under the same interest compounding (15% compounded quarterly)?

(b) What equal end-of-month deposit over twelve (12) years would accumulate the same amount at an interest rate of 15% compounded semiannually?

(c) What equal end-of-year deposit over twelve (12) years would accumulate the same amount at an interest rate of 15% per year compounded monthly?

Consider cash flows for the following investment projects (MARR = 13%):

<table>
<thead>
<tr>
<th>Period (n)</th>
<th>Project A</th>
<th>Project B</th>
<th>Project C</th>
<th>Project D</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-4500</td>
<td>-5000</td>
<td>-3500</td>
<td>-4000</td>
</tr>
<tr>
<td>1</td>
<td>1800</td>
<td>3000</td>
<td>1500</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1800</td>
<td>2000</td>
<td>1800</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>1800</td>
<td>1000</td>
<td>2100</td>
<td>5500</td>
</tr>
</tbody>
</table>

Suppose that projects are mutually exclusive. Which project would you select based on AE criterion?

You purchased an equipment for $45 000 to use in your own business. You do not expect the equipment to have a positive salvage or trade-in value after the anticipated 7-year life. For book depreciation purposes, you want depreciation cost ($D_n$) and book value ($B_n$) schedules for the following methods:

a) Straight-line depreciation method (SL),

b) 150% Declining-Balance (DB) method,

c) Double Declining-Balance (DDB) method

Hint: Use switching to reach salvage value $S=0$ for part (b) and (c)
1. Given: Single cash flows yearly (K=1); \( r = 15\% \), M=4 (quarterly); C=M/K=4/1=4
\[
i_p = (1 + \frac{r}{M})^C - 1 = (1 + \frac{0.15}{4})^4 - 1 = 0.1587 \text{ (15.87\% per year)}
\]
\[
P_{\text{given}} = $1000(P/A, 15.87\%, 3) + $2000(P/A, 15.87\%, 3)(P/F, 15.87\%, 3) \\
+ $3000(P/A, 15.87\%, 3)(P/F, 15.87\%, 6) \\
+ $4000(P/A, 15.87\%, 3)(P/F, 15.87\%, 9)
\]
\[
P_{\text{given}} = $1000 \left[ \frac{(1 + 0.1587)^3 - 1}{0.1587(1 + 0.1587)^3} \right] + $2000 \left[ \frac{(1 + 0.1587)^3 - 1}{0.1587(1 + 0.1587)^3} \right] \left( \frac{1}{1 + 0.1587^3} \right) \\
+ $3000 \left[ \frac{(1 + 0.1587)^3 - 1}{0.1587(1 + 0.1587)^3} \right] \left( \frac{1}{1 + 0.1587^3} \right) \\
+ $4000 \left[ \frac{(1 + 0.1587)^3 - 1}{0.1587(1 + 0.1587)^3} \right] \left( \frac{1}{1 + 0.1587^3} \right)
\]
\[P_{\text{given}} = $10325.58
\]

a) Find \( A_Q \) uniform equivalent quarterly cash flow under the same interest compounding
K=4; \( r = 15\% \), M=4 (quarterly); C=M/K=1 \( (i_p = i_c) \)
\[
i_p = (1 + \frac{r}{M})^C - 1 = (1 + \frac{0.15}{4})^4 - 1 = 0.0375 \text{ (3.75\% per quarter)}.
\]
\[
P_{\text{given}} = A_Q(P/A, 3.75\%, 48) \\
$10325.58 = A_Q \left[ \frac{(1 + 0.0375)^{48} - 1}{0.0375(1 + 0.0375)^{48}} \right]
\]
\[A_Q = $466.99
\]

b) Find \( A_M \) uniform equivalent monthly cash flow at the interest rate 15\% compounded semiannually
K=12; \( r = 15\% \), M=2 (semiannually); C=M/K=2/12=1/6=0.1667
\[
i_p = (1 + \frac{r}{M})^C - 1 = (1 + \frac{0.15}{2})^{0.1667} - 1 = 0.0121 \text{ (1.21\% per payment period (per month)).}
\]
\[
P_{\text{given}} = A_M(P/A, 1.21\%, 144) \\
$10325.58 = A_M \left[ \frac{(1 + 0.0121)^{144} - 1}{0.0121(1 + 0.0121)^{144}} \right]
\]
\[A_M = $151.80
\]

c) Find \( A \) (annuity) uniform equivalent yearly cash flow at the interest rate 15\% compounded monthly
K=1; \( r = 15\% \), M=12; C=M/K=12
\[
i_p = (1 + \frac{r}{M})^C - 1 = (1 + \frac{0.15}{12})^{12} - 1 = 0.1608 \text{ (16.08\% effective annual interest rate)}
\]
\[
P_{\text{given}} = A(P/A, 16.08\%, 12) \\
$10325.58 = A \left[ \frac{(1 + 0.1608)^{12} - 1}{0.1608(1 + 0.1608)^{12}} \right]
\]
\[A_Q = $1993.40
\]

2.
AE(13\%) = $4500(A/P, 13\%, 3) + $1800(P/A, 13\%, 3)(A/P, 13\%, 3) \\
= $4500(0.4235) + $1800 \\
= -$105.75 <0 (Interest factors were taken from Park Table/p.888)

AE(13\%) = $5000(A/P, 13\%, 3) + $3000(P/A, 13\%, 3)(A/P, 13\%, 3) \\
= $5000(0.4235) + $3000 - $1000(0.9187) \\
= -$36.20 <0 (Interest factors were taken from Park Table/p.888)
AE(13%)c=- $3500(A/P,13%,3) + $1500(P/A,13%,3)(A/P,13%,3) \\
+ $300(P/G,13%,3)(A/P,13%,3) \\
= - $3500(0.4235) + $1500 + $300(0.9187) \\
= $293.36 >0 (Interest factors were taken from Park Table/p.888)

AE(13%)d= - $4000(A/P,13%,3) + $5500(P/F,13%,3)(A/P,13%,3) \\
= - $4000(0.4235) + $5500(0.6931)(0.4235) \\
= $79.60 <0 (Interest factors were taken from Park Table/p.888)

Select Project C (AE>0)

3. 

a) 

\[
D_{SL} = \frac{I-nD}{N} = \frac{\$45000-0}{7} = \$6428.57
\]

\[
B_n = I - nD = \$45000 - n(\$6428.57)
\]

\[
B_N = I - ND = \$45000 - 7(\$6428.57) = 0 = S
\]

\[
\alpha = \left(\frac{1}{N}\right)\left(Multiplier\right) = \left(\frac{1}{7}\right)(1.50) = 0.2143
\]

\[
D_n = \alpha B_{n-1}; \quad B_{n-1} = I(1 - \alpha)^{n-1}
\]

\[
B_N = I(1 - \alpha)^N = \$45000(1 - 0.2143)^7 = 8317.83 > S
\]

use switching to SL at the beginning of year 4 (s.Table below)

b) 

\[
\alpha = \left(\frac{1}{N}\right)\left(Multiplier\right) = \left(\frac{1}{7}\right)(2.0) = 0.2857
\]

\[
D_n = \alpha B_{n-1}; \quad B_{n-1} = I(1 - \alpha)^{n-1}
\]

\[
B_N = I(1 - \alpha)^N = \$45000(1 - 0.2857)^7 = 4269.50 > S
\]

use switching to SL at the beginning of year 5 (s.Table below)

From the corresponding depreciation formulas of the methods given above and the using switching to SL for part (b) and (c) at the beginning of year 4, the results are tabulated as follows:

<table>
<thead>
<tr>
<th>n</th>
<th>SL</th>
<th>DB (α=0.2143)</th>
<th>DDB (α=0.2857)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_n$ ($)</td>
<td>$B_n$ ($)</td>
<td>$D_n$ ($)</td>
</tr>
<tr>
<td>0</td>
<td>45000</td>
<td>45000</td>
<td>45000</td>
</tr>
<tr>
<td>1</td>
<td>6428.57</td>
<td>38571.43</td>
<td>9643.50</td>
</tr>
<tr>
<td>2</td>
<td>6428.57</td>
<td>32142.86</td>
<td>7576.90</td>
</tr>
<tr>
<td>3</td>
<td>6428.57</td>
<td>25714.29</td>
<td>5953.17</td>
</tr>
<tr>
<td>4</td>
<td>6428.57</td>
<td>19285.72</td>
<td>4677.40 &lt; (5456.61=21826.43/4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Switch to SL</td>
</tr>
<tr>
<td>5</td>
<td>6428.57</td>
<td>12857.15</td>
<td>5456.61</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6428.57</td>
<td>6428.57</td>
<td>5456.61</td>
</tr>
<tr>
<td>7</td>
<td>6428.57</td>
<td>0</td>
<td>5456.61</td>
</tr>
</tbody>
</table>