Question: The saturation concentration of dissolved oxygen in water as a function of temperature at a chloride concentration of 10 g/L is listed in a table as follows:

<table>
<thead>
<tr>
<th>$T$, °C</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C$, mg/L</td>
<td>12.9</td>
<td>11.3</td>
<td>10.1</td>
<td>9.03</td>
<td>8.17</td>
<td>7.46</td>
<td>6.85</td>
</tr>
</tbody>
</table>

Use

a) First-order Newton interpolating polynomial,
b) Second-order Newton interpolating polynomial

to estimate the dissolved oxygen level for $T = 18$ °C. What is your better estimate?

Recall: Newton Interpolating Polynomial;

$f_{n-1}(x) = b_1 + b_2(x-x_1) + \ldots + b_n(x-x_1)(x-x_2)\ldots(x-x_{n-1})$

$b_1 = f(x_1)$, $b_2 = f[x_2, x_1]$; $b_3 = f[x_3, x_2, x_1]$, \ldots; $b_{n-1} = f[x_{n-1}, x_{n-2}, \ldots, x_1]$; $b_n = f[x_n, x_{n-1}, \ldots, x_2, x_1]$

$f[x_j, x_j] = \frac{f(x_j) - f(x_j)}{x_j - x_j}$; $f[x_j, x_j, x_k] = \frac{f(x_j, x_j) - f(x_j, x_k)}{x_j - x_k}$

$f[x_n, x_{n-1}, \ldots, x_2, x_1] = \frac{f[x_n, x_{n-1}, \ldots, x_2] - f[x_{n-1}, x_{n-2}, \ldots, x_1]}{x_n - x_1}$
MATH 302 NUMERICAL METHODS SPRING 13-14 QUIZ SOLUTION

Date: 13. 08. 2014
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a) For first-order interpolation, the concentration of dissolved oxygen is given by

\[ f_i(T) = C(T) = b_1 + b_2(T - T_i) \]

Since we want to find the concentration of dissolved oxygen at \( T = 330 \, K \) we need to choose the two data points that are closest to \( T = 18 \, ^\circ C \) and that also bracket \( T = 18 \, ^\circ C \). These two points are as follows:

\[ T_1 = 20 \, ^\circ C; \quad f(T_1) = C(T_1) = 8.17 \]
\[ T_2 = 15 \, ^\circ C; \quad f(T_2) = C(T_2) = 9.03 \]

then

\[ b_1 = C(T_1) = 8.17 \]
\[ b_2 = f[T_2, T_1] = \frac{f(T_2) - f(T_1)}{T_2 - T_1} = \frac{9.03 - 8.17}{15 - 20} = \frac{0.86}{-5} = -0.172 \]

\[ f_1(T) = C(T) = 8.17 + (-0.172)(T - 20) = 8.17 - 0.172(T - 20) \]

Thus, the linear estimate is 8.514 mg/L at \( T = 18 \, ^\circ C \). The quadratic estimate is better estimate of the concentration of dissolved oxygen with the value of 8.496 mg/L.

b) For second-order (quadratic) interpolation, the concentration of dissolved oxygen is given by

\[ f_2(T) = C(T) = b_1 + b_2(T - T_1) + b_3(T - T_1)(T - T_2) \]

Since we want to find the concentration at \( T = 18 \, ^\circ C \), we need to choose the three data points that are closest to \( T = 18 \, ^\circ C \) and that also bracket \( T = 18 \, ^\circ C \). These three points are as follows:

\[ T_1 = 20 \, ^\circ C; \quad f(T_1) = C(T_1) = 8.17 \]
\[ T_2 = 15 \, ^\circ C; \quad f(T_2) = C(T_2) = 9.03 \]
\[ T_3 = 25 \, ^\circ C; \quad f(T_3) = C(T_3) = 7.46 \]

then

\[ b_1 = C(T_1) = 8.17 \]
\[ b_2 = f[T_2, T_1] = \frac{f(T_2) - f(T_1)}{T_2 - T_1} = \frac{9.03 - 8.17}{15 - 20} = \frac{0.86}{-5} = -0.172 \]

\[ b_3 = f[T_3, T_2, T_1] = \frac{f[T_3, T_2] - f[T_2, T_1]}{T_3 - T_1} = \frac{f(T_3) - f(T_2) - f(T_2) - f(T_1)}{T_3 - T_1} \]
\[ = \frac{7.46 - 9.03}{25 - 15} - (-0.172) = \frac{-0.157 + 0.172}{5} = 3 \times 10^{-3} \]

\[ f_2(T) = C(T) = 8.17 - 0.172(T - 20) + 3 \times 10^{-3}(T - 20)(T - 15) \]
\[ f_2(18 \, ^\circ C) = C(18 \, ^\circ C) = 8.17 - 0.172(18 - 20) + 3 \times 10^{-3}(18 - 20)(18 - 15) \]
\[ = 8.514 + 3 \times 10^{-3}(18 - 20)(18 - 15) = 8.514 - 0.018 = 8.496 \]

Thus, the second-order (quadratic) estimate is better estimate of the concentration of dissolved oxygen with the value of 8.496 mg/L to the level of significant digits provided in the original data.