1. (35p) The following data show the number of hours spent watching television for 14 randomly selected sophomores attending a college in a country:

<table>
<thead>
<tr>
<th>Hours of Television Viewed Weekly</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5</td>
</tr>
<tr>
<td>5.4</td>
</tr>
</tbody>
</table>

a) Compute the mean, median and mode for these data.
b) Indicate whether the data are skewed or symmetrical.
c) Develop a box and whisker plot for these data.
d) Compute the sample variance, standard deviation and coefficient of variation.

2. (30p) A company that translates Chinese books into English is currently testing a computer-based translation service. Since Chinese symbols are difficult to translate, this company assumes the computer program will make some errors, but then so do human translators. The computer error rate is supposed to be an average of 3 per 400 words of translation. Suppose you randomly select a 1200-word passage. Assuming that the Poisson distribution applies, if the computer error rate is actually 3 errors per 400 words.
a) Determine the probability that no errors will be found?
b) Calculate the probability that more than 14 errors will be found.
c) Find the probability that fewer than 9 errors will be found.
d) What is the probability of finding at least 15 errors?
e) Calculate the probability that at most 10 errors will be found.

3. (35p) a) An airline wanted to determine if customers would be interested in paying a $10 flat fee for unlimited Internet access during long-haul flights. From a random sample of 200 customers, 125 indicated that they would be willing to pay this fee. Using this survey data, determine the 99% confidence interval estimate for the population proportion of the airline’s customers who would be prepared to pay this fee for Internet use.
b) Based on these sample results, a statistician computed a confidence interval extending from 0.55 to 0.70 for the population proportion. What is the confidence level of this interval?

**Confidence interval for the population proportion ($P$):**

$$\hat{p} - z_{a/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} < P < \hat{p} + z_{a/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

**Sample standard deviation ($s$) and coefficient of variation (CV):**

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n - 1}}; \quad CV = \frac{s}{\bar{x}} \times 100\%$$
a) sort the data from low to high:

<table>
<thead>
<tr>
<th>i</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>5.4</td>
<td>6.6</td>
<td>7.5</td>
<td>7.5</td>
<td>7.8</td>
<td>8.0</td>
<td>8.5</td>
</tr>
<tr>
<td>i</td>
<td>8.9</td>
<td>10.3</td>
<td>11.5</td>
<td>12</td>
<td>12.2</td>
<td>13</td>
<td>14.4</td>
</tr>
</tbody>
</table>

a) Mean:

\[
\bar{X} = \frac{\sum_{i=1}^{14} X_i}{n} = \frac{133.6}{14} = 9.5 \text{ h}
\]

The median is the 50th percentile. To determine the location index for the median (2nd quartile \( p = 50 \)) we do the following:

The index is

\[
i = \frac{p}{100} (n) = \frac{50}{100} (14) = 7 \text{ (integer)}
\]

Since the index, 7, is an integer, the 2nd quartile is determined by finding the average of the 7th and 8th values from the lower end of the sorted data. This is:

\[
Median = Q_2 = \frac{8.5 + 8.9}{2} = 8.7 \text{ h}
\]

The mode is the most frequently occurring value (7.5 h, two times)

b) \text{Mean} \neq \text{Median; Mean} > \text{Median (right skewed data – not symmetrical)}

c) To determine the location index for the 1st quartile \( p = 25 \)) we do the following:

The index is

\[
i = \frac{p}{100} (n) = \frac{25}{100} (14) = 3.5 \text{ (decimal value – round up to 4)}
\]

Since the index, 3.5, is a decimal value, the 1st quartile is determined by rounding up the index \( i \) to the next value of 4. The 1st quartile is the 4th value from the lower end of the sorted data. This is:

\[Q_1 = 7.5 \text{ h}\]

The location index for the 3rd quartile is:

\[
i = \frac{p}{100} (n) = \frac{75}{100} (14) = 10.5 \text{ (decimal)}
\]

Since the index, 10.5, is a decimal value, the 3rd quartile is determined by rounding up the index \( i \) to the next value of 11. The 3rd quartile is the 11th value from the lower end of the sorted data. This is:

\[Q_3 = 12 \text{ h}\]

The box and whisker plot is shown below.

```
               |               |
               |               |
               |               |
Min------------Q_1-Q_2-X-----------Q_3-Max
               |               |
               |               |
               |               |
5.4 7.5 8.7 9.5 12 14.4
```
d)

<table>
<thead>
<tr>
<th>(i)</th>
<th>(X)</th>
<th>(X - \bar{X})</th>
<th>((X - \bar{X})^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.4</td>
<td>5.4-9.5=-4.1</td>
<td>16.81</td>
</tr>
<tr>
<td>2</td>
<td>6.6</td>
<td>6.6-9.5=-2.9</td>
<td>8.41</td>
</tr>
<tr>
<td>3</td>
<td>7.5</td>
<td>7.5-9.5=-2.0</td>
<td>4.00</td>
</tr>
<tr>
<td>4</td>
<td>7.5</td>
<td>7.5-9.5=-2.0</td>
<td>4.00</td>
</tr>
<tr>
<td>5</td>
<td>7.8</td>
<td>7.8-9.5=-1.7</td>
<td>2.89</td>
</tr>
<tr>
<td>6</td>
<td>8.0</td>
<td>8.0-9.5=-1.5</td>
<td>2.25</td>
</tr>
<tr>
<td>7</td>
<td>8.5</td>
<td>8.5-9.5=-1.0</td>
<td>1.00</td>
</tr>
<tr>
<td>8</td>
<td>8.9</td>
<td>8.9-9.5=-0.6</td>
<td>0.36</td>
</tr>
<tr>
<td>9</td>
<td>10.3</td>
<td>10.3-9.5=0.8</td>
<td>0.64</td>
</tr>
<tr>
<td>10</td>
<td>11.5</td>
<td>11.5-9.5=2.0</td>
<td>4.00</td>
</tr>
<tr>
<td>11</td>
<td>12</td>
<td>12.0-9.5=2.5</td>
<td>6.25</td>
</tr>
<tr>
<td>12</td>
<td>12.2</td>
<td>12.2-9.5=2.7</td>
<td>7.29</td>
</tr>
<tr>
<td>13</td>
<td>13</td>
<td>13.0-9.5=3.5</td>
<td>12.25</td>
</tr>
<tr>
<td>14</td>
<td>14.4</td>
<td>14.4-9.5=4.9</td>
<td>24.01</td>
</tr>
</tbody>
</table>

\[ \sum (X - \bar{X})^2 = 94.16 \]

Sample variance:

\[ s^2 = \frac{\sum(X - \bar{X})^2}{n-1} = \frac{94.16}{14-1} = 7.24 \text{ } h^2 \]

Sample standard deviation:

\[ s = \sqrt{\frac{\sum(X - \bar{X})^2}{n-1}} = \sqrt{7.24} = 2.69 \text{ } h \]

Sample coefficient of variation (CV):

\[ CV = \frac{s}{\bar{X}} \times 100\% = \frac{2.69 \text{ } h}{9.5 \text{ } h} \times 100\% = 28.3\% \]

2.

a) This probability can be obtained using the individual Poisson probability distribution from Appendix Table 5 with the mean arrival rate of success as follows:

\[ \text{mean arrival rate, } \lambda = \left( \frac{3 \text{ words}}{400 \text{ words}} \right) (1200 - \text{word passage}) = 9 \]

\[ P(X = 0 | \lambda = 9) = 0.0001 \text{ (read from Appendix Table 5)} \]

b)

\[ P(X > 14 | \lambda = 9) = 1 - P(X \leq 14) = 1 - F(X = 14) = 1 - 0.9585 = 0.0415 \]

(from Appendix Table 6-cumulative Poisson Probabilities table) or from Table 5 as follows (long way):

\[ P(X > 14 | \lambda = 9) = 1 - P(X \leq 14) \]

\[ = 1 - [(P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10) + P(X = 11) + P(X = 12) + P(X = 13) + P(X = 14))] \]

\[ = 1 - (0.0001 + 0.0011 + 0.0050 + 0.0150 + 0.0337 + 0.0607 + 0.0911 + 0.1171 + 0.1318 + 0.1318 + 0.1186 + 0.0970 + 0.0728 + 0.0504 + 0.0324) = 1 - 0.9586 = 0.0414 \]
c) \[ P(X < 9|\lambda = 9) = P(X \leq 8|\lambda = 9) = F(X = 8|\lambda = 9) = 0.4557 \]  
(from Appendix Table 6-cumulative Poisson Probabilities)

d) \[ P(X \geq 15|\lambda = 9) = 1 - P(X \leq 14) = 1 - F(X = 14) = 1 - 0.9585 = 0.0415 \]  
(from Appendix Table 6-cumulative Poisson Probabilities table)

It was found the same probability with part (b)-equivalency

e) \[ P(X \leq 10|\lambda = 9) = F(X = 10|\lambda = 9) = 0.7060 \]  
(from Appendix Table 6-cumulative Poisson Probabilities)

3. 
a) Given: \[ n = 200, x = 125, \hat{p} = \frac{125}{200} = 0.625; \ CL = 1 - \alpha = 0.99 \ (99\%) \]
\[ \alpha = 1 - 0.99 = 0.01 \]
\[ \frac{\alpha}{2} = 0.005 \]
\[ z_{0.005} = 2.58 \ (corresponding \ to \ F(z) = 0.995 \ from \ Table \ 1) \]

Substitute in the formula given as follows to find 95% CI for the population proportion:
\[ \hat{p} - z_{a/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} < P < \hat{p} + z_{a/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \]
\[ 0.625 - 2.58 \sqrt{\frac{0.625(1 - 0.625)}{200}} < P < 0.625 + 2.58 \sqrt{\frac{0.625(1 - 0.625)}{200}} \]
\[ 0.625 - 0.0883 < P < 0.625 + 0.0883 \]
\[ 0.537 < P < 0.713 \]

or equivalently
\[ \hat{p} \mp z_{a/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = \hat{p} \mp Margin \ of \ Error = 0.625 \mp 0.0883 \]

b) Margin of Error (ME) can be found from the interval given:
\[ 2(ME) = 2 \left[ z_{a/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \right] = 0.70 - 0.55 = 0.15 \]
\[ 0.075 = z_{a/2} \sqrt{\frac{0.625(1 - 0.625)}{200}} \rightarrow z_{a/2} = 2.19 \rightarrow F(z) = 0.9857 \ (Table \ 1) \]
\[ \frac{\alpha}{2} = 1 - 0.9857 = 0.0143 \rightarrow \alpha = 2(0.0143) = 0.0286 \]
\[ CL = 1 - \alpha = 1 - 0.0286 = 0.9714 \ (97.14\%) \]