1. (35p) A cell phone company is studying the number of minutes used by clients in a particular cell phone rate plan. A random sample of 14 clients showed the following number of minutes used last month:

<table>
<thead>
<tr>
<th>Minutes</th>
<th>90</th>
<th>77</th>
<th>94</th>
<th>89</th>
<th>119</th>
<th>92</th>
<th>112</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>91</td>
<td>110</td>
<td>92</td>
<td>100</td>
<td>113</td>
<td>120</td>
<td>83</td>
</tr>
</tbody>
</table>

a) Compute the mean, median and mode for these data.
b) Indicate whether the data are skewed or symmetrical.
c) Develop a box and whisker plot for these data.
d) Compute the sample variance and standard deviation

2. (30p) A sample of executives were surveyed about loyalty to their company. One of the question was, ‘If you were given an offer by another company equal to or slightly better than your present position, would you remain with the company or take the other position?’. The responses of the 200 executives in the survey were cross-classified with their length of service with the company as follows:

<table>
<thead>
<tr>
<th>Loyalty</th>
<th>Less than 1 Year</th>
<th>1-5 Years</th>
<th>6-10 Years</th>
<th>More than 10 Years</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Would remain</td>
<td>10</td>
<td>30</td>
<td>5</td>
<td>75</td>
<td>120</td>
</tr>
<tr>
<td>Would not remain</td>
<td>25</td>
<td>15</td>
<td>10</td>
<td>30</td>
<td>80</td>
</tr>
<tr>
<td>Total</td>
<td>35</td>
<td>45</td>
<td>15</td>
<td>105</td>
<td>200</td>
</tr>
</tbody>
</table>

a) What is the probability of randomly selecting an executive who is loyal to the company (would remain) and who has more than 10 years of service?
b) What is the probability of randomly selecting an executive who would remain with the company or has less than 1 year of experience?

3. (35p) A study concluded that 50% of front seat occupants used seat belts. That means that both occupants of the front seat were using their seat belts. Suppose we decide to compare that information with current usage. We select a sample of 12 vehicles.
a) What is the probability the front seat occupants in exactly 7 of the 12 vehicles selected are wearing seat belts?
b) What is the probability the front seat occupants in at least 7 of the 12 vehicles selected are wearing seat belts?
c) What is the probability the front seat occupants in at most 7 of the 12 vehicles selected are wearing seat belts?
a) sort the data from low to high:

<table>
<thead>
<tr>
<th>i</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>77</td>
<td>83</td>
<td>89</td>
<td>90</td>
<td>91</td>
<td>92</td>
<td>92</td>
</tr>
<tr>
<td>i</td>
<td>94</td>
<td>100</td>
<td>110</td>
<td>112</td>
<td>113</td>
<td>119</td>
<td>120</td>
</tr>
</tbody>
</table>

a) Mean:

\[
\bar{X} = \frac{\sum_{i=1}^{14} X_i}{n} = \frac{1382}{14} = 98.7 \text{ min}
\]

The median is the 50th percentile. To determine the location index for the median (2nd quartile \( p = 50 \)) we do the following:

The index is

\[
i = \frac{p}{100} (n) = \frac{50}{100} (14) = 7 \text{ (integer)}
\]

Since the index, 7, is an integer, the 2nd quartile is determined by finding the average of the 7th and 8th values from the lower end of the sorted data. This is:

\[
Median = Q_2 = \frac{92 + 94}{2} = 93
\]

The mode is the most frequently occurring value (92, two times)

b) \( Mean \neq Median; \text{ Mean } > \text{ Median (right skewed data – not symmetrical)} \)

c) To determine the location index for the 1\(^{st}\) quartile \((p = 25)\) we do the following:

The index is

\[
i = \frac{p}{100} (n) = \frac{25}{100} (14) = 3.5 \text{ (decimal value – round up to 4)}
\]

Since the index, 3.5, is a decimal value, the 1\(^{st}\) quartile is determined by rounding up the index \( i \) to the next value of 4. The 1\(^{st}\) quartile is the 4th value from the lower end of the sorted data. This is:

\[Q_1 = 90\]

The location index for the 3\(^{rd}\) quartile is:

\[
i = \frac{p}{100} (n) = \frac{75}{100} (14) = 10.5 \text{ (decimal)}
\]

Since the index, 10.5, is a decimal value, the 3\(^{rd}\) quartile is determined by rounding up the index \( i \) to the next value of 11. The 3\(^{rd}\) quartile is the 11th value from the lower end of the sorted data. This is:

\[Q_3 = 112\]

The box and whisker plot is shown below.

```
---  ---  ---  ---  ---  ---  ---
```

```
77   90   93   112  120
```
d)

\[
\begin{array}{c|c|c|c}
 i & X_i & X_i - \bar{X} & (X_i - \bar{X})^2 \\
\hline
 1 & 77 & -21.7 & 470.89 \\
 2 & 83 & -15.7 & 246.49 \\
 3 & 89 & -9.7 & 94.09 \\
 4 & 90 & -8.7 & 75.69 \\
 5 & 91 & -7.7 & 59.29 \\
 6 & 92 & -6.7 & 44.89 \\
 7 & 92 & -6.7 & 44.89 \\
 8 & 94 & -4.7 & 22.09 \\
 9 & 100 & 1.3 & 1.69 \\
10 & 110 & 11.3 & 127.69 \\
11 & 112 & 13.3 & 176.89 \\
12 & 113 & 14.3 & 204.49 \\
13 & 119 & 20.3 & 412.09 \\
14 & 120 & 21.3 & 453.69 \\
\end{array}
\]

\[
\sum (X_i - \bar{X})^2 = 2434.86
\]

Sample variance:

\[
s^2 = \frac{\sum (X_i - \bar{X})^2}{n-1} = \frac{2434.86}{14-1} = 187.30 \text{ min}^2
\]

Sample standard deviation:

\[
s = \sqrt{\frac{\sum (X_i - \bar{X})^2}{n-1}} = \sqrt{187.30} = 13.69 \text{ min}
\]

2.

<table>
<thead>
<tr>
<th>Loyalty</th>
<th>Less than 1 Year (B_1)</th>
<th>1-5 Years (B_2)</th>
<th>6-10 Years (B_3)</th>
<th>More than 10 Years (B_4)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Would remain (A_1)</td>
<td>10</td>
<td>30</td>
<td>5</td>
<td>75</td>
<td>120</td>
</tr>
<tr>
<td>Would not remain (A_2)</td>
<td>25</td>
<td>15</td>
<td>10</td>
<td>30</td>
<td>80</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>35</strong></td>
<td><strong>45</strong></td>
<td><strong>15</strong></td>
<td><strong>105</strong></td>
<td><strong>200</strong></td>
</tr>
</tbody>
</table>

a) Two events (called A_1 and B_4) occur at the same time-the executive would remain with the company, and he or she has more than 10 years of service.

\[
P(A_1 \cap B_4) = \frac{75}{200} = 0.375 \text{ (from the crosstable or contingency table given)}
\]

or from the conditional probability formula as follows:

\[
P(A_1 \cap B_4) = P(A_1)P(B_4|A_1) = \left(\frac{120}{200}\right)\left(\frac{75}{120}\right) = \frac{75}{200} = 0.375
\]

b) To find the probability of selecting an executive who would remain with the company (event A_1) or has less than 1 year of experience (event B_1), we use general rule of addition as follows:

\[
P(A_1 \cup B_1) = P(A_1) + P(B_1) - P(A_1 \cap B_1) = \left(\frac{120}{200}\right) + \left(\frac{35}{200}\right) - \left(\frac{10}{200}\right) = \frac{145}{200} = 0.725
\]

So the likelihood that a selected executive would either remain with the company or has been with the company less than 1 year is 0.725.
3.

a) This probability can be obtained using the individual Binomial probability distribution from Appendix Table 2 with the probability of success $P = 0.5$ as follows:

$$P(x = 7|n = 12 \text{ and } P = 0.5) = 0.1934 \text{ (read from Appendix Table 2)}$$

b) $P(x \geq 7|n = 12 \text{ and } P = 0.5) = 1 - P(x \leq 6) = 1 - F(x \leq 6) = 1 - 0.613 = 0.387$

(from Appendix Table 3-cumulative Binomial Probability table) or from Table 2 as follows:

$$P(x \geq 7|n = 12 \text{ and } P = 0.5) = P(x = 7) + P(x = 8) + P(x = 9) + P(x = 10) + P(x = 11) + P(x = 12)$$

$$= 0.1934 + 0.1208 + 0.0537 + 0.0161 + 0.0029 + 0.0002 = 0.3871$$

c) $P(X \leq 7|n = 12 \text{ and } P = 0.5) = F(X \leq 7) = 0.806$

(from Appendix Table 3-cumulative Binomial Probability table)