1. (30 p) A random sample of 1556 people in country $A$ were asked to respond to this statement: ‘Increased world trade can increase our per capita prosperity.’ Of these sample members, 38.4% agreed with the statement. When the same statement was presented to a random sample of 1108 people in country $B$, 52% agreed. Test the null hypothesis at 1% significance level that the population proportions agreeing with this statement were the same in the two countries against the alternative that a higher proportion agreed in country $B$.

**Hint:** Use your appendix flow chart (Figure 10.6) with common proportion value $P_0$ of unknown population proportions $P_A$ and $P_B$. $P_0$ can be estimated by a pooled estimator defined as follows:

$$\hat{P}_0 = \frac{n_A\hat{P}_A + n_B\hat{P}_B}{n_A + n_B}$$

2. (35 p) Determine an equation to predict metabolic rate in unit Watt (Joule/s) as a function of mass in unit kg based on the following data:

<table>
<thead>
<tr>
<th>Living Creature</th>
<th>Cow</th>
<th>Human</th>
<th>Sheep</th>
<th>Hen</th>
<th>Rat</th>
<th>Dove</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass, kg</td>
<td>400</td>
<td>70</td>
<td>45</td>
<td>2</td>
<td>0.3</td>
<td>0.16</td>
</tr>
<tr>
<td>Metabolic rate, Watt</td>
<td>270</td>
<td>82</td>
<td>50</td>
<td>4.8</td>
<td>1.45</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Use power model with logarithmic transformations to fit the data by using least-squares regression.

$$b_1 = \frac{n\sum\sum(x_iy_i)-\sum x_i\sum x_i y_i}{n\sum x_i^2-(\sum x_i)^2}; b_0 = \bar{y} - b_1\bar{x}; \text{Power Model: MetRate} = \beta_0 (\text{Mass})^{\beta_1}$$

3. (35 p) Should all university students be required to own a laptop computer? Although this policy does exist at many universities, the added expense for students at small private schools should be considered. One business school recently surveyed its students to determine their reaction to this possible policy. The responses are given in the following table along with students’ majors as contingency table:

<table>
<thead>
<tr>
<th>Major</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accounting</td>
<td>68</td>
<td>42</td>
</tr>
<tr>
<td>Finance</td>
<td>40</td>
<td>15</td>
</tr>
<tr>
<td>Management</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>Marketing</td>
<td>30</td>
<td>25</td>
</tr>
</tbody>
</table>

Do the data indicate that there is an association between one’s major and the response to this question at 10% significance level? Formulas: $E_{ij} = \frac{RiCj}{n}$ for $(i = 1,2, ... r; j = 1,2, ... c)$

$$Re_the_H_0 \text{ if } \chi^2 = \sum\sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}} > \chi^2_{(r-1)(c-1),\alpha}$$
STAT 204 STATISTICS II SPRING 11-12 FINAL EXAM SOLUTIONS

1. First identify the null and alternative hypothesis. Use $P_A$ and $P_B$ to denote the population proportion of the people who agree with the statement in Country $A$ and $B$ respectively as follows:

   $$H_0: P_A - P_B \geq 0$$
   $$H_1: P_A - P_B < 0$$

   The decision rule is to reject $H_0$ and accept $H_1$ if

   \[ \Delta \hat{p} < \Delta \hat{p}_{\text{crit}} \]
   \[ \hat{p}_A - \hat{p}_B < -z_\alpha \sigma_{\Delta \hat{p}} \]
   \[ z = \frac{\hat{p}_A - \hat{p}_B}{\sigma_{\Delta \hat{p}}} < -z_\alpha \]

   \[ z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\frac{p_A(1 - p_A)}{n_A} + \frac{p_B(1 - p_B)}{n_B}}} < -z_\alpha \]

   The estimate of common proportion $P_0$ of unknown population proportions $P_A$ and $P_B$ can be found by a pooled estimator defined in question sheet as follows:

   $$\hat{p}_0 = \frac{n_A \hat{p}_A + n_B \hat{p}_B}{n_A + n_B}$$

   The data for this problem are as follows:

   $$n_A = 1556, \hat{p}_A = 0.384, n_B = 1108, \hat{p}_B = 0.52$$

   Now substitute these values into the formula to find the test statistic, rounding to two decimal places.

   $$\hat{p}_0 = \frac{(1556)(0.384) + (1108)(0.52)}{1556 + 1108} = 0.44$$

   $$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\frac{\hat{p}_0(1 - \hat{p}_0)}{n_A} + \frac{\hat{p}_0(1 - \hat{p}_0)}{n_B}}} = \frac{0.384 - 0.52}{\sqrt{(0.44)(1 - 0.44) + (0.44)(1 - 0.44)}} = -6.97$$

   $$-z_{0.01} = -2.33 \ (\text{Read from Appendix Table 1, p. 816})$$

   Compare the test statistic to $-z_{0.01}$

   $$z < -z_{0.01}$$
   $$-6.97 < -2.33$$

   Reject $H_0: P_A - P_B \geq 0$.

   Conclusion: There is sufficient evidence to conclude that a higher proportion agreed in country $B$ with this statement ($P_A - P_B < 0$).

2. Power Model: \( \text{MetRate} = \beta_0 (\text{Mass})^{\beta_1} \)

   Logarithmic transformation to linearize power model will give:

   \[ \log_{10}(\text{MetRate}) = \log_{10}\beta_0 + \beta_1 \log_{10}(\text{Mass}) \]

   Assuming

   \[ Y = \log_{10}(\text{MetRate}); \]
We get
\[ Y = b_0 + b_1X \]
This is a linear relationship between \( Y \) and \( X \).

\[
b_1 = \frac{n \sum^n X_i Y_i - \sum^n X_i \sum^n Y_i}{n \sum^n X_i^2 - (\sum^n X_i)^2}
\]
\[
b_0 = \bar{Y} - b_1 \bar{X}
\]

Table. Summations of data to calculate least-squares parameters of the power model.

<table>
<thead>
<tr>
<th>( i )</th>
<th>((Mass)_i)</th>
<th>((MetRate)_i)</th>
<th>( X_i = \log_{10}(Mass)_i )</th>
<th>( Y_i = \log_{10}(MetRate)_i )</th>
<th>( X_iY_i )</th>
<th>( X_i^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400</td>
<td>270</td>
<td>2.6021</td>
<td>2.4314</td>
<td>6.3268</td>
<td>6.7709</td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>82</td>
<td>1.8451</td>
<td>1.9138</td>
<td>3.5312</td>
<td>3.4044</td>
</tr>
<tr>
<td>3</td>
<td>45</td>
<td>50</td>
<td>1.6532</td>
<td>1.6990</td>
<td>2.8088</td>
<td>2.7331</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>4.8</td>
<td>0.3010</td>
<td>0.6812</td>
<td>0.2050</td>
<td>0.0906</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
<td>1.45</td>
<td>-0.5229</td>
<td>0.1614</td>
<td>-0.0844</td>
<td>0.2734</td>
</tr>
<tr>
<td>6</td>
<td>0.16</td>
<td>0.97</td>
<td>-0.7959</td>
<td>-0.0132</td>
<td>0.0105</td>
<td>0.6335</td>
</tr>
<tr>
<td>( \Sigma )</td>
<td></td>
<td></td>
<td>5.0826</td>
<td>6.8736</td>
<td>12.7979</td>
<td>13.9059</td>
</tr>
</tbody>
</table>

From the least square equations given above, we have

\[
b_1 = \frac{6(12.7979) - (5.0826)(6.8736)}{6(13.9059) - (5.0826)^2} = 0.7266
\]
\[
b_0 = \frac{6.8736}{6} - 0.7266 \frac{5.0826}{6} = 0.5301
\]

Since \( b_0 = 10^{b_0}; b_1 = \beta_1 \)

\[
\beta_0 = 10^{0.5301} = 3.3892; b_1 = \beta_1 = 0.7266
\]

Fitting equation to the data: \( MetRate = 3.3892(Mass)^{0.7266} \)

3. \( H_0: \) No association exists between students' s opinion on laptops and their major
\( H_1: \) There is an association between students' s opinion on laptops and their major

Chi-Square Test:

<table>
<thead>
<tr>
<th>Major</th>
<th>Laptop Required?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
</tr>
<tr>
<td>Accounting</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>( (66) )</td>
</tr>
<tr>
<td>Finance</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>( (33) )</td>
</tr>
<tr>
<td>Management</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>( (66) )</td>
</tr>
<tr>
<td>Marketing</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>( (33) )</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>198</td>
</tr>
</tbody>
</table>
Expected number of observations in each cell of the contingency table (in parentheses as bold and italic) under $H_0$ are found by using the formula given in the question and substituted in the definition of Chi-Square $\chi^2$ variable definition as follows:

$$E_{11} = \frac{R_1 C_1}{n} = \frac{(110)(198)}{330} = 66$$
$$E_{12} = \frac{R_1 C_2}{n} = \frac{(110)(132)}{330} = 44$$
$$\vdots$$
$$E_{41} = \frac{R_4 C_1}{n} = \frac{(55)(198)}{330} = 33$$
$$E_{42} = \frac{R_4 C_2}{n} = \frac{(55)(132)}{330} = 22$$

$$\chi^2 = \frac{(O_{11} - E_{11})^2}{E_{11}} + \frac{(O_{12} - E_{12})^2}{E_{12}} + \ldots + \frac{(O_{41} - E_{41})^2}{E_{41}} + \frac{(O_{42} - E_{42})^2}{E_{42}}$$

$$\chi^2 = \frac{(68 - 66)^2}{66} + \frac{(42 - 44)^2}{44} + \ldots + \frac{(30 - 33)^2}{33} + \frac{(25 - 22)^2}{22} = 5.909$$

The degree of freedom are:

$$(r - 1)(c - 1) = (4 - 1)(2 - 1) = 3, \text{ there are 4 rows, 2 columns in the table}$$

From Table 7 in the appendix, we find critical Chi-Square variable as follows:

$$\chi^2_{(r-1)(c-1),\alpha} = \chi^2_{3,0.10} = 6.25$$

$$Reject H_0 \text{ if } \chi^2 = 5.909 > \chi^2_{3,0.10} = 6.25$$

No. Therefore, do not reject $H_0$ of no association at the 10% significance level. There is sufficient evidence at the 10% significance level that no association between students’ opinion on laptops and their majors exists.