1. (35 p) A random sample of six salespersons that attended a motivational course on sales techniques was monitored three months before and three months after the course. The table shows the values of sales (in thousands of dollars) generated by these six salespersons in the two periods:

<table>
<thead>
<tr>
<th>Salesperson</th>
<th>Before the Course</th>
<th>After the Course</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>206</td>
<td>227</td>
</tr>
<tr>
<td>2</td>
<td>286</td>
<td>286</td>
</tr>
<tr>
<td>3</td>
<td>209</td>
<td>201</td>
</tr>
<tr>
<td>4</td>
<td>332</td>
<td>344</td>
</tr>
<tr>
<td>5</td>
<td>167</td>
<td>195</td>
</tr>
<tr>
<td>6</td>
<td>204</td>
<td>190</td>
</tr>
</tbody>
</table>

Assume that the population distributions are normal. Find a 99% confidence interval for the difference between the two population means.

2. (35 p) A company that receives shipments of batteries tests a random sample of sixteen of them before agreeing to take a shipment. The company is concerned that the true mean lifetime for all batteries in the shipment should be at least 50 hours. From past experience it is safe to conclude that the population distribution of lifetimes is normal with a standard deviation of 3 hours. For one particular shipment the mean lifetime for a sample of sixteen batteries was 47.6 hours.

a) Test at the 5% significance level the null hypothesis that the population mean lifetime is at least 50 hours.

b) Find the power of a 5%-level test when the true mean lifetime of batteries is 49.5 hours.

3. (30 p) A medical research group is investigating what differences might exist between two pain-killing drugs, Azerlieve (Drug A) and Zynumbic (Drug Z). The researchers have already established there is no difference between the two drugs in terms of the average amount of time required before they take effect. However, they are also interested in knowing if there is any difference between the variability of time until pain relief occurs. A random sample of 24 patients using Azerlieve and 31 patients using Zynumbic yielded the following results:

<table>
<thead>
<tr>
<th>Azerlieve (Drug A)</th>
<th>Zynumbic (Drug Z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n_A = 24$</td>
<td>$n_Z = 31$</td>
</tr>
<tr>
<td>$s_A = 37.5$ seconds</td>
<td>$s_Z = 41.3$ seconds</td>
</tr>
</tbody>
</table>

At the 0.10 level of significance, test if there is evidence of a difference between $\sigma_A^2$ and $\sigma_Z^2$ using a two-sided (two-tail) alternative hypothesis.

$$d - t_{\alpha/2} \frac{s_d}{\sqrt{n}} < \mu_d < d + t_{\alpha/2} \frac{s_d}{\sqrt{n}}$$

$$s_d = \sqrt{\frac{\sum (d_i - \bar{d})^2}{n - 1}}; F = \frac{s_x^2}{s_y^2}$$
STAT 204 STATISTICS II SPRING 11C12 MIDTERM EXAM SOLUTIONS

1. To find a 100(1-α)% confidence interval for the difference between means

\( (\mu_d = \mu_{\text{after}} - \mu_{\text{before}}) \), first calculate \( \bar{d} \), the sample mean difference. Calculate the differences as follows:

<table>
<thead>
<tr>
<th>After</th>
<th>Before</th>
<th>Difference</th>
<th>((d_i - \bar{d}))</th>
<th>((d_i - \bar{d})^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>227</td>
<td>206</td>
<td>21</td>
<td>(21 - 6.5)</td>
<td>210.25</td>
</tr>
<tr>
<td>286</td>
<td>286</td>
<td>0</td>
<td>(0 - 6.5)</td>
<td>42.25</td>
</tr>
<tr>
<td>201</td>
<td>209</td>
<td>-8</td>
<td>(-8 - 6.5)</td>
<td>210.25</td>
</tr>
<tr>
<td>344</td>
<td>332</td>
<td>12</td>
<td>(12 - 6.5)</td>
<td>30.25</td>
</tr>
<tr>
<td>195</td>
<td>167</td>
<td>28</td>
<td>(28 - 6.5)</td>
<td>462.25</td>
</tr>
<tr>
<td>190</td>
<td>204</td>
<td>-14</td>
<td>(-14 - 6.5)</td>
<td>420.25</td>
</tr>
</tbody>
</table>

Determine the mean of the differences:

\[ \bar{d} = \frac{21 + 0 + (-8) + 12 + 28 + (-14)}{6} = 6.5 \]

Calculate the sample standard deviation as follows:

\[ s_d = \sqrt{\frac{\sum (d_i - \bar{d})^2}{n - 1}} = \sqrt{\frac{210.25 + 42.25 + 210.25 + 30.25 + 462.25 + 420.25}{6 - 1}} = 16.5861 \]

Now determine \( t_{n-1, \alpha/2} \), the number for which \( P \left( t_{n-1} > t_{n-1, \frac{\alpha}{2}} \right) = \frac{\alpha}{2} \) with (n-1) degrees of freedom. Use table of cutoff points for Student’s t distribution (Table 8) as follows:

For a 99% confidence level, \( \alpha = 0.01 \), \( \frac{\alpha}{2} = 0.005 \), \( n - 1 = 5 \)

\( t_{5, 0.005} = 4.032 \) (Read from Table 8, p.844).

Use the formula given at the bottom of the question sheet to calculate confidence interval:

\[ \bar{d} - t_{v, \alpha/2} \frac{s_d}{\sqrt{n}} < \mu_d < \bar{d} + t_{v, \alpha/2} \frac{s_d}{\sqrt{n}} \]

Substitute the known values and simplify:

\[ 6.5 - 4.032 \frac{16.5861}{\sqrt{6}} < \mu_d < 6.5 + 4.032 \frac{16.5861}{\sqrt{6}} \]

\[ -20.80 < \mu_d < 33.80 \]

2. a) First identify the null and alternative hypothesis

\( H_0: \mu \geq 50 \)
\( H_1: \mu < 50 \)

To test \( H_0: \mu \geq \mu_0 \) against the alternative \( H_1: \mu < \mu_0 \) the decision rule is to reject \( H_0 \) and accept \( H_1 \) if

\[ \bar{x} < x_{\text{crit}} \quad (\text{From Appendix Fig.9.10}) \]

\[ x_{\text{crit}} = \mu_0 - z_{\alpha} \sigma_x \]

\[ z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} < -z_{\alpha} \]
\[ \bar{x} = 47.6, \mu_0 = 50, \sigma = 3, n = 16 \]

Now substitute these values into the formula to find the test statistic, rounding to two decimal places.

\[ z = \frac{47.6 - 50}{\frac{3}{\sqrt{16}}} = -3.20 \]

\[ -z_{0.05} = -1.645 \text{ (Read from Appendix Table 1, p. 815)} \]

Compare the test statistic to \(-z_{0.05}\)

\[ z < -z_{0.05} \]

\[-3.20 < -1.645 \]

Reject \(H_0: \mu \geq 50\).

Conclusion: There is sufficient evidence that the mean lifetime of the batteries is less than 50 hours.

**b)** The probability of a Type II error, \(\beta\), for any value of the population mean defined by the alternative hypothesis is the probability that the sample mean will be in the nonrejection region for the null hypothesis. The power of the test is \(1 - \beta\).

Begin by determining the probability of a Type II error for the true mean \(\mu^* = 49.5\) h

First determine \(\overline{x}_c\):

\[ x_{crit} = \mu_0 - z_{\alpha} \sigma_{\overline{x}} = \mu_0 - z_{\alpha} \left( \frac{\sigma}{\sqrt{n}} \right) = 50 - 1.645 \left( \frac{3}{\sqrt{16}} \right) = 48.766 \]

\[ \beta = P(\bar{x} \geq 48.766 | \mu = 49.5) = P \left( z \geq \frac{48.766 - 49.5}{\frac{3}{\sqrt{16}}} \right) = P(z \geq -0.98) \]

\[ P(z \geq -0.98) = 1 - P(z \leq -0.98) = 1 - P(z \geq 0.98) = 1 - [1 - P(z \leq 0.98)] \]

\[ = P(z \leq 0.98) = 0.8365 \text{ (read from Appendix Table 1, p. 815)} \]

\[ Power = 1 - \beta = 1 - 0.8365 = 0.1635 \]

3. First identify the null and alternative hypothesis:

\[ H_0: \sigma_x^2 = \sigma_y^2 \]

\[ H_1: \sigma_x^2 \neq \sigma_y^2 \text{ (two sided)} \]

Use F test statistic with the definition given at the bottom of the question sheet:

\[ F = \frac{s_x^2}{s_y^2}, \text{use } s_x^2 = s_Z^2 \text{ (larger sample variance in numerator)}; s_y^2 = s_A^2 \]

\[ F = \frac{(41.3)^2}{(37.5)^2} = 1.2129 \]

Given: \(\alpha = 0.10\), \(n_A = 24, n_Z = 31\)

Now determine \(F_{n_Z-1,n_A-1,\alpha/2}\) such that \(P(F_{n_Z-1,n_A-1} > F_{n_Z-1,n_A-1,\alpha/2}) = \frac{\alpha}{2}\) with the numerator degrees of freedom \((n_Z-1)\) and denominator degrees of freedom \(n_A - 1\). Use table of cutoff points for the F distribution (Table 9) as follows:

For a 10% significance level, \(\alpha = 0.10, \frac{\alpha}{2} = 0.05, (n_Z - 1) = 30, (n_A - 1) = 23\)
\[ F_{n_2-1,n_A-1,\alpha/2} = F_{30,23,0.05} = 1.96 \ (Read\ from\ Table\ 9,p.846) \]

Compare with the test statistic F. Reject \( H_0 \), if the computed F test statistic (1.2129) is greater than \( F_{30,23,0.05} \) (1.96).

Since 1.2129 < 1.96, do not reject the null hypothesis and conclude as follows:

Conclusion: There is no evidence of differences in the variability of the effect time for the 2 drugs.