1. **(35 p)** A manufacturer knows that the number of items produced per hour by machine A and by machine B are normally distributed with a standard deviation of 8.4 items for machine A and a standard deviation of 11.3 items for machine B. The mean hourly amount produced by machine A for a random sample of 40 hours was 130 units; the mean hourly amount produced by machine B for a random sample of 36 hours was 120 units. Find the 95% confidence interval for the difference in mean parts produced per hour by these two machines.

2. **(35 p)** A pharmaceutical manufacturer is concerned that the impurity concentration in pills should not exceed 3%. It is known that from a particular production run impurity concentrations follow a normal distribution with a standard deviation of 0.4%. A random sample of 64 pills from a production run was checked, and the sample mean impurity concentration was found to be 3.07%.

   a) Test at the 5% significance level the null hypothesis that the population mean impurity concentration is 3% against the alternative that it is more than 3%.

   b) Find the $p$-value for this test.

   c) In the context of this problem, explain why a one-sided alternative hypothesis is more appropriate than a two-sided alternative.

3. **(30 p)** A university research team was studying the relationship between idea generation by groups with and without a moderator. For a random sample of four groups with a moderator, the mean number of ideas generated per group was 78.0, and the standard deviation was 24.4. For a random sample of four groups without a moderator, the mean number of ideas generated was 63.5, and the standard deviation was 20.2. Test the assumption that the two population variances were equal against the alternative that the population variance is higher for groups with a moderator at the 5% significance level.
1. Let $X$ = machine A and $Y$ = machine B

Given:
\[
\bar{x} = 130, \sigma_x = 8.4, n_x = 40; \bar{y} = 120, \sigma_y = 11.3, n_y = 36
\]

Determine the mean of the differences:
\[
\bar{d} = \bar{x} - \bar{y} = 130 - 120 = 10
\]

To find the 95% confidence interval for the difference in means parts produced per hour by the two machines, use the formula given at the bottom of the question sheet as follows:

\[
\bar{d} - z_{\alpha/2} \sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}} < \mu_d < \bar{d} + z_{\alpha/2} \sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}
\]

To determine $z_{\alpha/2}$, use Table 1 as follows:

For a 95% confidence level, $\alpha = 0.05, \frac{\alpha}{2} = 0.025, F(z) = 1 - 0.025 = 0.975$

\[
z_{\alpha/2} = 1.96 \quad (Read \ from \ Table \ 1, \ Appendix \ Tables/p. \ 816)
\]

Substitute the known values in the formula given (STAT204 Guidelines C-1/03/04/14) as follows:

\[
10 - 1.96 \sqrt{\frac{(8.4)^2}{40} + \frac{(11.3)^2}{36}} < \mu_d < 10 + 1.96 \sqrt{\frac{(8.4)^2}{40} + \frac{(11.3)^2}{36}}
\]

\[
10 - 4.5169 < \mu_d < 10 + 4.5169
\]

or
\[
5.4831 < \mu_d < 14.5169
\]

or
\[
10 \pm 4.5169
\]

2. a) First identify the null and alternative hypothesis

$H_0: \mu = 3\%$

$H_1: \mu > 3\%$

To test $H_0: \mu = \mu_0$ against the alternative $H_1: \mu > \mu_0$ the decision rule is to reject $H_0$ and accept $H_1$ if

\[
x > x_{crit} (\sigma^2 \text{known, Type 2 Hypothesis, From Appendix Fig.9.10})
\]

\[
x_{crit} = \mu_0 + z_\alpha \sigma_x
\]

\[
z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}} > z_\alpha \quad (Guidelines \ D - 1)
\]

\[
\bar{x} = 3.07\%, \mu_0 = 3\%, \sigma = 0.4\%, n = 64
\]

Now substitute these values into the formula to find the test statistic, rounding to two decimal places.

\[
z = \frac{3.07\% - 3\%}{0.4\% / \sqrt{64}} = 1.4
\]
\[ \alpha = 5\%, z_{0.05} = 1.645 \ (Read \ from \ Appendix \ Table \ 1, p.815) \]

Compare the test statistic to \( z_{0.05} \)

\[ \text{If } z > z_{0.05}, \text{Reject } H_0 \]

\[ 1.4 < 1.645 \]

Do Not Reject \( H_0 \) at the 5% significance level: \( \mu = 3\% \).

Conclusion: There is sufficient evidence that the mean impurity concentration in pills is 3% or less than 3%.

b) \[ p - value = P(\ z > 1.4) = 1 - F(1.4) = 1 - 0.9192 = 0.0808 > \alpha \]

c) A one-sided alternative is more appropriate since we are not interested in detecting possible low levels of impurity, only high levels of impurity in pills.

3. First identify the null and alternative hypothesis:

\[ H_0: \sigma_x^2 \leq \sigma_y^2 \]

\[ H_1: \sigma_x^2 > \sigma_y^2 \ (one\ -\ sided) \]

Use F test statistic with the definition given at the bottom of the question sheet:

\[ F = \frac{s_y^2}{s_x^2}, \text{ (larger sample variance in numerator)}; \text{ Moderator Group as } s_x^2 \]

\[ F = \frac{(24.4)^2}{(20.2)^2} = 1.46 \]

Given: \( \alpha = 0.05, \ n_x = 4 \) (Moderator group), \( n_y = 4 \) (No moderator group)

Now determine \( F_{n_x-1,n_y-1,\alpha} \) such that \( P \left( F_{n_x-1,n_y-1} \right) = \alpha \) with the numerator degrees of freedom \( (n_x-1) \) and denominator degrees of freedom \( n_y - 1 \). Use table of cutoff points for the F distribution (Appendix Table 9/p.845) as follows:

For a 5% significance level, \( \alpha = 0.05 \) (one - sided), \( (n_x - 1) = 3, (n_y - 1) = 3 \)

\[ F_{n_x-1,n_y-1,\alpha} = F_{3,3,0.05} = 9.28 \ (Read \ from \ Table \ 9, p.845) \]

Compare with the test statistic \( F \). Reject \( H_0 \), if the computed F test statistic (1.46) is greater than \( F_{3,3,0.05}(9.28) \).

Since 1.46 < 9.28, do not reject the null hypothesis and conclude as follows:

Conclusion: There is no evidence of higher variances in groups with a moderator.