1. (30 p) A researcher intends to estimate the effect of a drug on the scores of human subjects performing a task of psychomotor coordination. The members of a random sample of 9 subjects were given the drug prior to testing. The mean score in this group was 9.78, and the sample variance was 17.64. An independent random sample of 10 subjects was used as control group and given a placebo prior to testing. The mean score in this control group was 15.10, and the sample variance was 27.01. Assuming that the population distributions are normal with equal variances, find a 90% confidence interval for the difference between the population mean scores.

2. (40 p) Of a random sample of 199 auditors, 104 indicated some measure of agreement with this statement: Cash flow is an important indication of profitability.

a) Test at the 10% significance level against a two-sided alternative, the null hypothesis that 50% of the members of this population would agree with this statement.

b) Find and interpret the p-value of this test.

c) Find the probability of accepting false null hypothesis with a 10%-level test if, in fact, 60% of all auditors agree that cash flow is an important indicator of profitability.

3. (30 p) In light of a number of recent large-corporation bankruptcies, auditors are becoming increasingly concerned about the possibility of fraud. Auditors might be helped in determining the chances of fraud if they carefully measure cash flow. To evaluate this possibility, samples of midlevel auditors from Certified Public Accountant (CPA) firms were presented with cash-flow information from a fraud case, and they were asked to indicate the chance of material fraud on a scale from 0 to 100. A random sample of 41 auditors used the cash-flow information. Their mean assessment was 36.21, and the sample standard deviation was 22.93. For an independent random sample of 31 auditors not using the cash-flow information, the sample mean and standard deviation were respectively 47.56 and 27.56. Test the assumption that population variances for assessments of the chance of material fraud were the same for auditors using cash-flow information as for auditors not using cash-flow information against a two-sided alternative hypothesis at the 10% significance level.
1. Use Appendix Diagram C-2. Let subscript 1 = Group 1 and 2 = Group 2 (Control Group)

Given:
\[ \bar{x}_1 = 9.78, s^2_1 = 17.64, n_1 = 9; \bar{x}_2 = 15.10, s^2_2 = 27.01, n_2 = 10 \]

Determine the mean of the differences:
\[ d = \bar{x}_1 - \bar{x}_2 = 9.78 - 15.10 = -5.32 \]

To find the 90% confidence interval for the difference between the population mean scores, use the formula given at the Appendix Diagram C-2 as follows:
\[ \bar{d} - t_{n_1+n_2-2,\alpha/2} \frac{s_p^2}{n_1} + \frac{s_p^2}{n_2} < \mu_d < \bar{d} + t_{n_1+n_2-2,\alpha/2} \frac{s_p^2}{n_1} + \frac{s_p^2}{n_2} \]

\[ s_p = \sqrt{\frac{(n_1 - 1)s^2_1 + (n_2 - 1)s^2_2}{n_1 + n_2 - 2}} = \sqrt{\frac{(9 - 1)(17.64) + (10 - 1)(27.01)}{9 + 10 - 2}} = 4.754 \]

To determine \( t_{n_1+n_2-2,\alpha/2} \), use Table 8 as follows:

For a 90% confidence level, \( \alpha = 0.10 \), \( \alpha/2 = 0.05 \), \( t_{n_1+n_2-2,\alpha/2} = t_{9+10-2,0.10/2} = t_{17,0.05} = 1.74 \)

(Substitute the known values in the formula given in the diagram C-2 as follows:

\[ -5.32 - (1.74)(4.754) \sqrt{\frac{1}{9} + \frac{1}{10}} < \mu_d < -5.32 + (1.74)(4.754) \sqrt{\frac{1}{9} + \frac{1}{10}} \]

\[ -5.32 - 3.8007 < \mu_d < -5.32 + 3.8007 \]

or

\[ -9.1207 < \mu_d < -1.5193 \]

2. a) Use Diagram D-4 or Figure 9.11 with the first hypothesis type as follows:

\( H_0: P = P_0 = 0.50 \) (given 50%)

\( H_1: P \neq P_0 \) (two-sided)

To test \( H_0: P = P_0 \) against the alternative \( H_1: P \neq P_0 \) the decision rule is to reject \( H_0 \) and accept \( H_1 \) if

\[ z = \frac{\hat{p} - P_0}{\sigma_{\hat{p}}} > z_{\alpha/2} \text{ or } z = \frac{\hat{p} - P_0}{\sigma_{\hat{p}}} < -z_{\alpha/2} \]

(Guidelines D - 4; Requirement \( nP(1 - P) \geq 5 \) is fulfilled, \( P = 0.5, n = 199 \))

\( \sigma_{\hat{p}} \) calculated with the null hypothesis population proportion

\[ \hat{p} = \frac{104}{199} = 0.5226; \sigma_{\hat{p}} = \sqrt{\frac{P_0(1 - P_0)}{n}} = \sqrt{\frac{0.50(1 - 0.50)}{199}} = 0.03544 \]
Now substitute these values into the formula to find the z-test statistic, rounding to two decimal places.

\[
z = \frac{0.5226 - 0.50}{0.03544} = 0.6376
\]

\[
\alpha = 10\%, z_{\alpha/2} = z_{0.10/2} = z_{0.05} = 1.645 \text{ (corresponding to } F(z) = 0.95 \text{ Read from Appendix Table 1, p.815)}
\]

Compare the test statistic to \( z_{0.05} \)

If \( z > z_{0.05}, \text{Reject } H_0 \)

\[
0.6376 < 1.645
\]

Do Not Reject \( H_0 \) at the 10% significance level: \( P = 50\% \).

Conclusion: There is insufficient evidence to reject null hypothesis.

b) 
\[
p - value = 2P(z > 0.6376) = 2[1 - F(0.64)] = 2(1 - 0.7389) = 2(0.2611)
\]

\[
= 0.5222 > \alpha \text{ (10%)}
\]

Conclusion: There is insufficient evidence to reject null hypothesis at 10% significance level.

c) True proportion given as follows:

\[
P_1 = 0.60 \text{ (60%)}
\]

The probability of accepting false null hypothesis is \( \beta \) (Type II Error) and can be found as follows:

\[
\text{critical values of } \hat{p} = P_0 \pm z_{\alpha/2}\sigma_{\hat{p}} = 0.50 \pm 1.645(0.03544);
\]

\[
\text{upper critical value } \hat{p}_{ucrit} = 0.558, \text{ lower critical value } \hat{p}_{lcrit} = 0.442
\]

\[
\beta = P(0.442 < \hat{p} < 0.558| P_1 = 0.60)
\]

\[
= P(-4.55 < z < -1.2094) = F(z = -1.2094) - F(z = -4.55)
\]

\[
= 1 - F(z = 1.2094) - [1 - F(z = 4.55)] = 1 - 0.8869 - [1 - 1]
\]

\[
= 0.1131
\]

3. Use the Diagram E-5. First identify the null and alternative hypothesis:

\[
H_0: \sigma_x^2 = \sigma_y^2
\]

\[
H_1: \sigma_x^2 \neq \sigma_y^2 \text{ (two - sided)}
\]

Use F test statistic with the definition given at the diagram E-5:

Auditors not using cash flow (CF) with larger variance as \( s_1^2 \)

\[
F = \frac{s_1^2}{s_2^2} = \frac{(27.56)^2}{(22.93)^2} = 1.44
\]

Given:

\( \alpha = 0.10, \ n_1 = 31 \) (Auditor group not using CF), \( n_2 = 41 \) (Auditor group used CF)

Now determine \( F_{n_1-1,n_2-1,\alpha/2} \) such that \( P(F_{n_1-1,n_2-1} > F_{n_1-1,n_2-1,\alpha/2}) = \alpha/2 \) with the numerator degrees of freedom \( (n_1-1) \) and denominator degrees of freedom \( n_2 - 1 \). Use table of cutoff points for the F distribution (Appendix Table 9/p.845) as follows:

For a 10% significance level, \( \alpha = 0.10, \alpha/2 = 0.05 \text{ (two - sided)}, \)
\( (n_1 - 1) = 30, (n_2 - 1) = 40 \)

\[
F_{n_1-1,n_2-1,a/2} = F_{30,40,0.05} = 1.74 \text{ (Read from Table 9, p. 846)}
\]

Compare with the test statistic F. Reject \( H_0 \), if the computed F test statistic (1.44) is greater than \( F_{30,40,0.05} \).

Since 1.44 < 1.74, **Do not reject the null hypothesis** and conclude as follows:

Conclusion: There is no evidence of different variance with the auditor group not using CF information.