1. (30 p) Recent business graduates currently employed in full-time positions were surveyed. Family backgrounds were self-classified as relatively high or low socioeconomic status. For a random sample of 16 high-socioeconomic status recent business graduates, the mean total compensation was $34500 and the sample standard deviation was $8520. For an independent random sample of 9 low-socioeconomic status recent business graduates, the mean total compensation was $31499 and the sample standard deviation was $7521. Find a 90% confidence interval for the difference between the two population means by assuming equal variances.

2. (40 p) A random sample of 802 supermarket shoppers determined that 378 shoppers preferred generic brand items.

a) Test at the 10% significance level the null hypothesis that at least one-half of all shoppers preferred generic brand items against the alternative that the population proportion is less than one-half.

b) Find and interpret the $p$-value of this test.

c) Find the probability of accepting false null hypothesis with a 10%-level test if, in fact, 45% of the supermarket shoppers preferred generic brands.

3. (30 p) The recent financial collapse has led to considerable concern about the information provided to potential investors. The government and many researchers have pointed out the need for increased regulation of financial offerings. The study in this exercise concerns the effect of sales forecasts on initial public offerings. Initial public offerings’ prospectuses were examined. In a random sample of 70 prospectuses in which sales forecasts were disclosed, the mean debt-to-equity ratio prior to the offering issue was 3.97, and the sample standard deviation was 6.14. For an independent random sample of 51 prospectuses in which sales earnings forecasts were not disclosed, the mean debt-to-equity ratio was 2.86, and the sample standard deviation was 4.29. By assuming both populations are normal with equal variances:

a) Test, against a two-sided alternative at the 10% significance level, the null hypothesis that population mean debt-to-equity ratios are the same for disclosers and nondisclosers of earning forecasts.

b) Test, against an one-sided alternative (right tail/upper tail) at the 1% significance level, the null hypothesis that population mean debt-to-equity ratios are equal for disclosers and nondisclosers or lower for disclosers than nondisclosers.
STAT 204 STATISTICS II
SPRING 15-16 MIDTERM EXAM SOLUTIONS

1. Use Appendix Diagram C-2. Let subscript 1 = High-socioeconomic status recent business graduates and 2 = Low-socioeconomic status recent business graduates.

Given:
\[
\bar{x}_1 = \$34500, s_1 = \$8520, n_1 = 16; \quad \bar{x}_2 = \$31499, s_2 = \$7521, n_2 = 9
\]

Determine the mean of the differences:
\[
d = \bar{x}_1 - \bar{x}_2 = \$34500 - \$31499 = \$3001
\]

To find the 90% confidence interval for the difference between the population mean compensations, use the formula given at the Appendix Diagram C-2 as follows:
\[
d - t_{n_1+n_2-2,\alpha/2} \sqrt{\frac{s^2_1}{n_1} + \frac{s^2_2}{n_2}} < \mu_d < d + t_{n_1+n_2-2,\alpha/2} \sqrt{\frac{s^2_1}{n_1} + \frac{s^2_2}{n_2}}
\]

\[
s_p = \sqrt{\frac{(n_1-1)s^2_1 + (n_2-1)s^2_2}{n_1 + n_2 - 2}} = \sqrt{\frac{(16-1)(\$8520)^2 + (9-1)(\$7521)^2}{16 + 9 - 2}} = \$8186.36
\]

To determine \( t_{n_1+n_2-2,\alpha/2} \), use Table 8 as follows:

For a 90% confidence level, \( 1 - \alpha = 0.90 \rightarrow \alpha = 0.10, \frac{\alpha}{2} = 0.05 \), \( t_{n_1+n_2-2,\alpha/2} = t_{16+9-2,0.10/2} = t_{23,0.05} = 1.714 \)

(Read from Table 8, Appendix Tables/p.844)

Substitute the known values in the formula given in the diagram C-2 as follows:

(STAT204 Guidelines to be used in the exams dated 03/04/14)

\[
\frac{1}{16} + \frac{1}{9} < \mu_d < \frac{1}{16} + \frac{1}{9}
\]

\[
\$3001 - (1.714)(\$8186.36) < \mu_d < \$3001 + (1.714)(\$8186.36)
\]

\[
\$3001 - 5846.43 < \mu_d < \$3001 + 5846.43
\]

\[
-2845.43 < \mu_d < 8847.43
\]

or

\[
\$3001 \mp 5846.43
\]

2. a) Use Diagram D-4 with the third hypothesis type at the right side of the rectangle of diagram as follows:

\( H_0: P \geq P_0 = 0.50 \) (given at least one – half of all shoppers, i.e. 50%)

\( H_A: P < P_0 \) (lower tail/one – tailed)

To test \( H_0: P \geq P_0 \) against the alternative \( H_A: P < P_0 \) the decision rule is to reject \( H_0 \) and accept \( H_A \) if

\[
z = \frac{\hat{p} - P_0}{\sigma_{\hat{p}}} < -z_{\alpha/2}
\]

(Guidelines D – 4; Requirement \( nP(1-P) \geq 5 \) is fulfilled, \( P = 0.5, n = 802 \))

(\( \sigma_{\hat{p}} \) calculated with the null hypothesis population proportion)
\[ \hat{\rho} = \frac{378}{802} = 0.471; \sigma_{\hat{\rho}} = \sqrt{\frac{P_0(1-P_0)}{n}} = \sqrt{\frac{0.50(1-0.50)}{802}} = 0.01766 \]

Now substitute these values into the formula to find the z-test statistic, rounding to two decimal places.

\[ z = \frac{0.471 - 0.50}{0.01766} = -1.64 \]

\( \alpha = 10\%, -z_{\alpha} = -z_{0.10} = -1.28 \) (corresponding to \( F(z) = 0.90 \) Read from Appendix Table 1, p. 815)

Compare the test statistic to \( -z_{0.10} \)

If \( z < -z_{0.10} \), Reject \( H_0 \)

\(-1.64 < -1.28 \)

Yes. Reject \( H_0 \) at the 10% significance level: \( P < 50\% \).

Conclusion: There is sufficient evidence to reject null hypothesis at the 10% significance level.

b) \( p-value = P(z < -1.64) = P(z > 1.64) = [1 - F(z = 1.64)] = (1 - 0.9495) = 0.0505 < \alpha (0.10 \rightarrow 10\%) \rightarrow \text{Reject } H_0 \)

Conclusion: There is sufficient evidence to reject null hypothesis at the 10% significance level.

c) True population proportion given as follows:

\( P_1 = 0.45 \) (45%)

The probability of accepting false null hypothesis is \( \beta \) (Type II Error) and can be found as follows:

\[ \text{critical value of } \hat{\rho} = P_0 - z_{\alpha} \sigma_{\hat{\rho}} = 0.50 - 1.28(0.01766); \]

\[ \beta = P(\hat{\rho} > 0.477|P_1 = 0.45) \]

\[ \beta = P \left( \frac{z > 0.477 - 0.45}{0.45(1 - 0.45)} \right) \sqrt{\frac{0.45(1 - 0.45)}{802}} \]

\[ \beta = P(z > 1.54) = 1 - F(z = 1.54) = 1 - 0.9382 = 0.0618 \]

The power of the test \( = 1 - \beta = 1 - 0.0618 = 0.9382 \)

3.a) Let \( 1= \) prospectuses in which sales forecasts were disclosed ; \( 2= \) prospectuses in which sales earnings forecasts were not disclosed

Assuming both populations are normal with equal variances, use Appendix/Guideline E-2.

First identify the null and alternative hypothesis

\( H_0: \mu_1 - \mu_2 = 0 \)

\( H_1: \mu_1 - \mu_2 \neq 0 \)

To test \( H_0: \mu_1 - \mu_2 = 0 \) against the two-sided alternative \( H_1: \mu_1 - \mu_2 \neq 0 \) the decision rule is to reject \( H_0 \) and accept \( H_1 \) if

\[ t > t_{n_1+n_2-2,\alpha/2} \text{ or } t < -t_{n_1+n_2-2,\alpha/2} \] (two – tail test, From Appendix Guidelines Diagram E – 2)
\[ t = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \]  

(Guidelines E - 2)

\[ s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}; \text{pooled standard deviation} \]

\[ \bar{x}_1 = 3.97, \bar{x}_2 = 2.86, n_1 = 70, n_2 = 51, s_1 = 6.14, s_2 = 4.29 \]

Now substitute these values into the formula to find the test statistic, rounding to three decimal places.

\[ s_p = \sqrt{\frac{(70 - 1)(6.14)^2 + (51 - 1)(4.29)^2}{70 + 51 - 2}} = 5.440; t = \frac{(3.97 - 2.86) - 0}{5.440 \sqrt{\frac{1}{70} + \frac{1}{51}}} = 1.108 \]

\[ \alpha = 0.10, t_{70+51-2,0.10/2} = t_{119,0.05} = t_{\infty,0.05} = 1.645 \]  

(Appendix Table 8, p. 844)

(For \( v \to \infty \) degree of freedom, \( t \) – value approaches to \( z \) value of 1.645)

\[ F(z) = 0.95 \text{ from Appendix Table 1, p. 815} \]

Compare the test statistic to \( t_{\infty,0.05} \)

\[ 1.108 > 1.645 \rightarrow \text{NO} \]

Do not Reject \( H_0 \) at the 10% significance level since 1.108 < 1.645

Conclusion: There is insufficient evidence that the population mean debt-to-equity ratios are not equal for disclosers and nondisclosers of earning forecasts at the 10% level of significance.

b) First identify the null and alternative hypothesis

\[ H_0: \mu_1 - \mu_2 \leq 0 \]
\[ H_1: \mu_1 - \mu_2 > 0 \]

\[ \bar{x}_1 = 3.97, \bar{x}_2 = 2.86, n_1 = 70, n_2 = 51, s_1 = 6.14, s_2 = 4.29 \]
\[ s_p = 5.440; t = 1.108 \]

\[ \alpha = 0.01, t_{70+51-2,0.01} = t_{119,0.005} = t_{\infty,0.01} = 2.326 \]  

(Appendix Table 8, p. 844)

Compare the test statistic to \( t_{\infty,0.01} \)

\[ 1.108 > 2.326 \rightarrow \text{NO} \]

Do not Reject \( H_0 \) also at the 1% significance level since 1.108 < 2.326

Conclusion: There is insufficient evidence that the population mean debt-to-equity ratios are higher for disclosers than for nondisclosers of earning forecasts.